November 2019

Exploring Teacher Candidates' Facilitation of Common Core Mathematical Practice Three

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Exploring Teacher Candidates' Facilitation of Common Core Mathematical Practice Three

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in Curriculum and Instruction
with a concentration in Elementary Education
Department of Teaching and Learning
College of Education
University of South Florida

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Date of Approval:
October 28, 2019

Keywords: Practice-based methods, clinically-rich teacher preparation, elementary mathematics

Dedication

This dissertation is dedicated to my family, especially my husband Rob, my children Bianca and Robby, my parents Mario and Carmelann, my brother Mario, my sister Maryangela, and friends that have become my family, for without them, this work would be meaningless.

Acknowledgments

I did not earn this degree on my own, with gratitude I'd like to share this accomplishment with those that provided the greatest support over the last four and a half years.

To my committee, Dr. Sarah van Ingen, you have been on the receiving end of all my insecure moments. Thank you for your patience, knowledge, and understanding. From my first semester, you supported my growth throughout my doctoral program. You always made me believe I was capable, and it is because of your support I have succeeded. I am truly honored to have had this opportunity with you.

Dr. Jenn Jacobs, Dr. Eugenia Vomvoridi-Ivanovich, and Dr. Janet Richards, your wisdom has become my inner voice and purpose as I move forward in the field of education. Thank you for sharing your time and knowledge with me.

Dr. Angie Hooser, I can still remember the day I met you. I did not know what a great inspiration, mentor, savior, and friend you would become. I cannot even begin to thank you for the support you have given me throughout this whole experience. I couldn't have asked for a better person to have with me on this adventure. I know our friendship will be forever.

To my family, my everything, without you I would not have accomplished this work. Rob, Bianca, and Robby thank you for allowing me to chase my dream, even if it did move us halfway across the country. You kept me going when I wanted to quit, you lifted me up when I fell and spoke words of encouragement when I doubted myself. Bianca and Robby, there were missed concerts, sports games, and outings, but I hope that you know I love you more than words. All I had to do was look into your eyes, listen to your laughter, or hold you to find strength. If you

have learned anything from this journey, I hope that you have learned to follow your dreams and that anything is possible with hard work and persistence. And yes, I'll be there in the morning when you get up and, in the afternoon when you get home!

To my husband, Rob, thank you for your unconditional love, goodness knows I needed it. I share the biggest part of this degree with you. I cannot even begin to list all the ways that you have supported me these last four and a half years. I could not have done this without your advocacy and validation. Your dedication to our family and work at home has not gone unnoticed or unappreciated. Thank you for encouraging, hugging, understanding, being patient, and listening. And for all that is good in the world, please stop asking me when I will be done!

To my mom and dad, words cannot express how much I love you. Thank you for always believing in me even when I did not believe in myself. I am truly blessed to have your unwavering, relentless love and encouragement.

To my brother and sister, thank you for understanding the need to pursue this degree. I know it was not easy to grasp what pulled me away from New York and our family. I love you both so very much.

To my dear cousins, Erin and Marybeth, thanks for being on speed dial (or should I say text). I am so fortunate to have your constant friendship, faith, and encouragement.

I could not have chosen better friends, colleagues, and peers to stand with me throughout this experience. To my loudest cheerleader and most exquisite friend, Deborah, I cannot express how much you mean to me. Thank you for your faith, for answering my phone calls, for talking to me for hours, for your letters, and for your unbridled enthusiasm for my success every second of every day. You had me at yoo-hoo.

Sarah Reals, my road trip co-pilot, thank you for being brave enough to start this journey with me. We share so many fun and incredible memories that I will cherish forever. I look forward to making many more. I would walk five hundred miles for you!

My dear friend Elyse, thank you for opening your home and giving me a family when I needed it most. Thank you for the mini-van express, drinking (lots) of coffee with me, proofreading countless papers, and celebrating each (small and large) moment of success! I look forward to seeing what our lives have in store for us. Our friendship is forever.

Annette, I am grateful for finding great friendship at the most unexpected time and place. You were one of my favorite distractions.

My doc student peers who have talked me down from the ledge more times than I can count. Especially, Denise and Joy, you are my confidants, cheerleaders, counselors, but most of all, my family!

And finally, my students who became my colleagues…may the passion to teach burn for a long time to come. Never stop pursuing to be better and know more than yesterday. Thank you for learning alongside me.

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Abstract

The Common Core State Standards in Mathematics (CCSSM, 2010), refer to eight distinct Mathematical Practices describing what *students* should be doing for optimal learning during mathematics lessons. Specifically, Mathematical Practice 3 (MP3), "construct viable arguments and critique the reasoning of others" (CCSSM, 2010, para 4), posits students who are proficient in mathematics are able to compare different solutions, distinguish correct and logically sound answers from those that are incorrect and then explain why the solution is incorrect (CCSSM, 2010). MP3 requires teachers to provide opportunities for students to engage in discussions beyond just "show and tell" talk to develop a deeper knowledge (Stein, Engle, Smith, & Hughes, 2008). There is limited literature identifying teaching practices or behaviors for facilitating MP3 in the elementary classroom and how TCs learn to facilitate these practices. This study addresses the gap in the literature related to teacher preparation. It provides a proof-of-concept example of how method courses and field placement link theory to practice and outlines potential learning trajectories of TCs who engage in these practices.

I used an exploratory multiple case study to gain a better understanding of the ways three TCs learn to facilitate MP3, "construct viable arguments and critique the reasoning of others" (CCSSM, 2010) across math methods courses and accompanying clinical internships in elementary classrooms. I explored the similarities and differences in the ways each TC planned and enacted teaching behaviors for the facilitation of MP3. The following research questions helped me come to understand how TCs learn to facilitate MP3: Within the context of a practice-

based methods course and an accompanying clinical internship placement in an elementary classroom, how do three TCs learn to facilitate Common Core MP3? How does the TC plan to facilitate MP3 in their clinical internship classrooms? How does the TC enact teaching behaviors for facilitating MP3 in their clinical internship classrooms? In what ways do TCs perceive supports and tensions within a practice-based methods course and clinical internship classrooms when facilitating MP3?

First, I explored each case as a separate entity, attending to the configurations within the case, followed by a comparative analysis across cases attending to interpretive synthesis and common themes (Miles, Huberman, & Saldana, 2014). I used three rounds of coding during individual data analysis to assign categories and themes from the data sources. I focused on and described what behaviors and actions TCs exhibited (rather than behaviors not exhibited) and provided evidence for how each facilitated MP3. Finally, a cross-case analysis compared cases and made sense of the commonalities and differences in TCs' facilitation.

I found the data supported a conceptual model, or trajectory, for understanding the ways TCs (and novice teachers) learn to facilitate MP3 and might apply to other Mathematical Practices or content areas.

Chapter One: Introduction

Teaching is more than simply, "helping others learn to do particular things" (Ball & Forzani, 2009, p. 498). Knowing how to teach is not knowledge one naturally acquires in everyday life. It involves specific professional knowledge that is very different from everyday activities. Ball and Forzani (2009) stated, "skillful teaching requires appropriately using and integrating specific moves and activities in particular cases and contexts, based on knowledge and understanding of one's pupils and on the application of professional judgment" (p. 497). Engaging in these actions with students effectively is complex and demanding work relying on what teachers know. What a teacher knows and does in the classroom greatly impacts what is taught and, ultimately, what students learn (Ball, Hill, & Bass, 2005; Hiebert & Grouws, 2007). Educators and those in education agree teachers' actions play the single most important role in the classroom.

Background

Learning the complex work of teaching ought to begin within teacher preparation programs as teacher education greatly influences future teachers' knowledge and behavior in the classroom (Darling-Hammond, 2006). However, there has been a persistent incongruity between teachers' educational preparation and the increasingly arduous demands of the classroom (Ravitch, 2013). There is also disagreement as to what successful teacher preparation might look like and how to prepare strong teacher candidates. Traditional teacher preparation programs have been grounded in learning about theory and applying theory as separate, consecutively addressed topics. Many describe this epistemology as a disconnected approach to learning where what is

being taught in campus courses (theory) is not directly applied to experiences in K-12 classrooms (Darling-Hammond, 2006; Zeichner, 2010) Additionally, with recent criticism regarding Teacher Candidates' (TC) insufficient amount of time within classrooms (Forzani, 2014), current reform efforts in teacher preparation focus on the process of applying theory (what is learned is courses) to practice (what is happening within K-12 classrooms) (Cochran-Smith & Zeichner, 2005).

As an answer to the criticism that university-based teacher preparation courses and K-12 classrooms are disconnected, teacher preparation programs have shifted to be more centered around experiences in clinical (classroom) settings (Zeichner, 2010). Clinically based teacher education programs, as defined by the National Council of Accreditation of Teacher Education's (NCATE) Blue Ribbon Report (2010), necessitate universities to partner with K-12 schools in an effort to bridge the theory to practice divide within teacher education programs. Clinically based programs use methods courses to embed clinical experiences and provide TCs opportunities to observe, rehearse, and enact teaching practices (Grossman et al., 2009). Even more recently, both the American Association of Colleges for Teacher Education's (AACTE) Clinical Practice Commission (2018) and the Association of Mathematics Teacher Educators (AMTE, 2017) acknowledged the importance of clinical settings for supporting teacher candidates in learning from using practices themselves and from the teaching of others.

Lampert (2004) defines *teaching practice* as beyond "what teachers do", but more specifically the intentional behavior enacted with students (p.2). Facilitating opportunities for TCs to learn teaching practice, requires teacher preparation curricula to identify specific instructional practices that have been referred to in the literature as core, high-leverage, or ambitious teaching practices (Ball, Thames, & Phelps, 2008; Ball & Forzani, 2009; Grossman, Hammerness, &McDonald, 2009; Lampert, 2010; Zeichner, 2012; Forzani, 2014). These

instructional, or pedagogical, practices of teaching are defined by Grossman, Hammerness, and McDonald (2009) as actions that occur frequently in teaching, can be implemented by TCs despite varying curriculum and instructional methods, allow TCs to learn about teaching and learning, uphold the complexity and integrity of teaching, are grounded in research, and can improve student learning.

A recent stream of research on practice-based teacher preparation focuses on methods of intentional learning, rehearsing, and enacting these content-specific instructional practices (Zeichner, 2012; Grossman, Hammerness & McDonald, 2009). Practice-based pedagogy prepares teachers in meaningful ways (Ball & Forzani, 2009; Grossman, Hammerness, & McDonald, 2009) by linking explicit connections from coursework to opportunities for the rehearsal of practices embedded in extended clinical internships (Darling-Hammond, 2006). Further, research has supported methods courses that strategically target instructional practices for teaching to prepare candidates for providing high-quality instruction (Jacobs & Spangler, 2017).

Rationale

Ball and Cohen (1999) argue university-based experiences in coursework are critical places for learning such professional practices. McDonald, Kazemi, & Kavannagh (2013) invite teacher educators to develop a shared and cumulative understanding of how TCs learn to teach within practice-based courses in order to develop their individual practice and progress the field of teacher education by,

> specifying a common language for specifying practice, which would facilitate the field's ability to engage in collective activity, identify and specify common pedagogies in

> teacher education; and address the perennial and persistent divides among university courses and between university coursework and clinical experiences (p. 2).

In response, teacher educators are exploring ways to define what they do around core instructional practices and share the work done with elementary TCs (e.g. Percy & Troyan, 2017; Kazemi, Ghousseini, Cunard, & Turrou, 2016). Teacher educators have found that engaging TCs in coached clinical internships (Campbell & Dunleavy, 2016), deliberately creating practice and rehearsal opportunities (Ghousseini & Herbst, 2016; 2017; Anthony, Hunter & Hunter, 2015), and reflecting on experiences (Bailey & Taylor, 2015) can support learning and development of core teaching practices.

Specifically, in the field of mathematics teacher preparation, there is a growing body of research focusing on core practices for teaching and methods for learning them (Ball & Forzani, 2009; McDonald, Kazemi, & Kavanaugh, 2013; Percy & Troyan, 2017). McDonald, Kazemi, and Kavanagh (2013) posited,

> by highlighting specific, routine aspects of teaching that demand the exercise of professional judgment and the creating of meaningful intellectual and social community for teachers, teacher educators, and students, core practices may offer teacher educators powerful tools for preparing teachers for the constant in-the-moment decision-making that the profession requires (p. 1).

Researchers are working to identify core practices for teaching elementary mathematics (e.g. Kazemi, Franke, & Lampert, 2009; Ball & Forzani, 2009; McDonald, Kazemi, & Kavanaugh, 2013) and embedding instructional activities which provide opportunities for TCs to enact and rehearse core practices (Kazemi, Ghousseini, Cunard, & Turrou, 2016; Lampert et al., 2013).

The National Council of Teachers of Mathematics (NCTM) (2014) defined eight effective core *Mathematical Teaching Practices* in their report titled Principles to Actions. The council stated, effective mathematics teaching "engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (p. 7). The eight Mathematical Teaching Practices highlighted by NCTM are; establish mathematics goals to focus learning, implement tasks that promote reasoning and problem solving, use and connect mathematical representations, facilitate meaningful mathematical discourse, pose purposeful questions, build procedural fluency from conceptual understanding, support productive struggle in learning mathematics, and elicit and use evidence of student learning.

Whereas NCTM's Mathematical Teaching Practices describe what *teachers* should be doing for effective mathematics instruction, The Common Core State Standards in Mathematics (CCSSM, 2010), refer to eight distinct Mathematical Practices describing what *students* should be doing during mathematics lessons. CCSSM (2010) states, students who are proficient in mathematics are able to compare different solutions, distinguish correct and logically sound answers from those that are incorrect and then explain why the solution is incorrect. The CCSSM Mathematical Practices are research-proven praxes resting on NCTM's (2000) set of five Process Standards and National Research Council's (2001) strands for mathematical proficiency. NCTM's five Process Standards describe the ways in which students develop and use mathematical knowledge and are considered essential elements for learning and teaching mathematics (NCTM, 2000). NRC's report, Adding It Up described the strands for mathematical proficiency as a "framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency" (p.116).

It is agreed in these documents written by policymakers and mathematics teacher education researchers that engaging students in reasoning, proving, augmentation, and justification opportunities are important for learning mathematics at a deeper level in elementary classrooms. NCTM (2000) stated, "reasoning and proof should be a consistent part of students' mathematical experience in prekindergarten through grade 12" (p. 56). Additionally, when describing Mathematical Practice 3 (MP3), construct viable arguments and critique the reasoning of others, CCSSM (2010) states students "proficient in mathematics should be able to justify their conclusions, communicate them to others, and respond to arguments of others" (p. 117). MP3 requires teachers to provide opportunities for students to engage in mathematical discussions beyond just "show and tell" talk to develop deeper mathematical knowledge (Stein, Engle, Smith, & Hughes, 2008). For teachers, this means they need to know how to engage their students in rich meaningful discourse around justification and proving which includes, encouraging responses to other students and using the ideas of others as a source to deepen understanding of mathematical procedures and concepts.

In an attempt to identify teaching behaviors that facilitate opportunities for students to engage in the CCSSM Mathematical Practices, Bostic, Matney, and Sondergeld (2017) created a "Look-Fors" tool for observing teachers' instructional behavior. According to Bostic, Matney and Sondergeld (2017) teachers facilitating MP3 might provide and facilitate opportunities for students to listen and discuss the solution strategies of others, listen and discuss alternative solutions strategies, and defend their ideas. In addition, teachers could ask higher-level questions that support students to back-up their ideas and think about other student(s) response(s). Also, teachers can present tasks that support students to think deeply about the mathematics they are studying which are related to proving events. Finally, teachers can provide students opportunities

to engage in proving events that support them in improving and revising their mathematical arguments.

There is limited literature identifying teaching practices or behaviors for facilitating MP3 and how TCs learn to facilitate these practices. Broadly, this study addresses the gap in the literature base related to research on practice-based teacher education that identifies; research-based core practices, designs of methods courses that link theory to practice, and learning trajectories of TCs who engage in these practices. To support the learning of complex practices of teaching, teacher educators "must be informed by what novice teachers do and find challenging in practice" (Ghousseini, 2015, p. 335). Through the lens of an elementary math teacher educator, I aimed to explore the ways TCs learn to facilitate MP3. The purpose of this dissertation study was to understand the ways TCs learn to enact the instructional practices needed to facilitate MP3 within the contexts of a practice-based mathematics methods course and clinical internships.

Activity Theory as a Theoretical Framework

Building on Vygotsky's theory of social-cultural theory of cognitive development, Wertsch (1981) and Leont'ev (1981) defined activity theory as grounded in the idea that people learn because of engagement in activity systems. Additionally, activity theory suggests that "conscious learning emerges from activity, not as a precursor to it" (Jonassen & Roher-Murphy, 1999, p. 62). Learning that occurs through an activity system is dependent on several components and relies on achieving certain goals in collaboration with others while being refereed by certain tools or instruments. Leont'ev (1981) described activity theory as having organized levels of an activity (activity system) with the components being a subject, objects, processes, and mediational tools.

Activity system. Further developing on Wertsch's (1981) and Leont'ev's (1981) ideas, Engestrom's (1993) depicted the structures within an activity system and emphasized how each influence another. Engestrom (1987) additionally described three principles for observing human behaviors within an activity system. First, an activity system is grounded in an intentional contextual situation. This context gives the activity system its meaning and connects a series of otherwise random events. Second, the elements of the activity system can be historically understood. How a person acts is grounded in her or his previous experiences and, as a result, there may be differences found within the various components of the system. Third, the conflicts or tensions within the system can be analyzed as a cause of disruption or change in that system. Disruptions can be identified by tracking and following disruptions at distinct points throughout the activity system. Further, studying interactivity within the system will reveal any invisible influences not initially presumed to play a role in the system.

The activity system components. Engestrom (1993) argued an activity system is composed of the subject, object, and instruments (tools) as depicted in Figure 1. These components integrated with subsystems of production, consumption, exchange, and distribution make up the unified activity system. According to Engestrom's (1993) model, the *subject* of the activity system is a person or people whose perspective is selected as the focus of study. The *object* signifies the space or material that is to be transformed or molded into the outcome. This transformation is aided by *tools* (internal, external, physical or symbolic) and a group of people who are a part of the *community* that shares in the work (*division of labor*) of the object. The *rules* denote norms, conventions, or regulations that are either implicit or explicit within the setting of the activity system. The activity system is continuously being reconstructed by the interactions of these components, and as such *tools* are being reformulated and *rules* are being obeyed or amended.

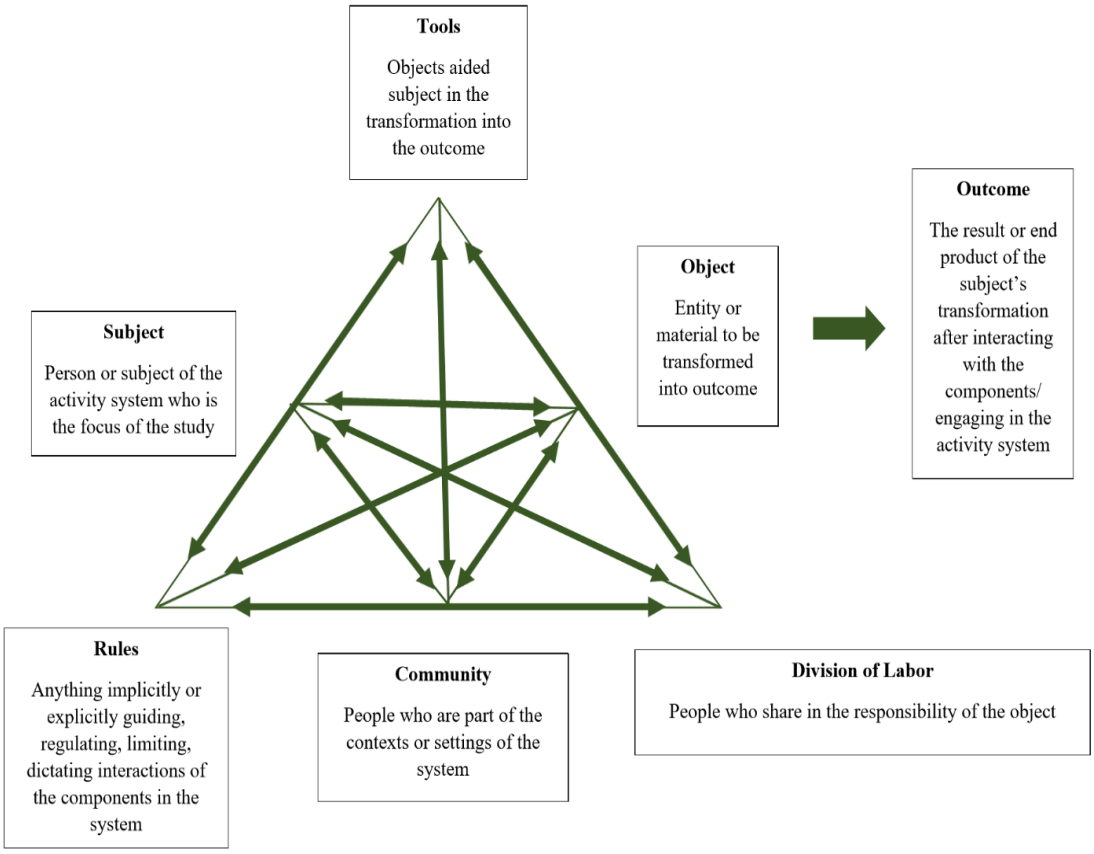

Figure 1. Model of Activity System Components

Historical and cultural settings in an activity system. Grossman and her colleagues (1999) emphasized the importance of the historical and cultural settings of the activity system. If we consider teaching as a cultural act being formed by those that engage in the activity (Stigler & Hiebert, 1998), then learning how to teach must happen within these settings and in collaboration with those that engage in it. However, how effective teaching is defined and what it looks like can differ a great deal from setting to setting. For a teacher educator, the ultimate goal for TCs is to have them take on the role of teacher and engage in effective teaching practices.

Appropriation. One of the key concepts of activity theory is appropriation (Wertsch, 1981; Leont'ye, 1981). Appropriation denotes the learning progression through which the subject

accepts and uses tools in social settings. For TCs, appropriation is a developmental process that allows them to adopt tools associated with and for teaching (Wertsch, 1981; Leont'ye, 1981). The factors contributing to appropriation can spur from the context or environment of the activity or the individualities of the learner, such as prior experiences as a student, personal goals and expectations, and beliefs and content knowledge. Through the appropriation of an activity system, TCs embrace and internalize the pedagogy used in teaching. The degree a subject may appropriate tools can be attributed to their prior knowledge about a situation, their values, and shared goals with experienced participants in the community (Wertsch, 1981).

Grossman et. al (1999) describe five degrees in which appropriation may or may not occur including lack of appropriation, appropriating a label, appropriating surface features, appropriating conceptual underpinnings, and achieving mastery. Lack of appropriation can happen for several reasons including not comprehending a concept, not having prior knowledge to understand a concept, or understanding a concept, but rejecting it for some reason. Knowing about a tool at a surface level (i.e. learning name of tool) describes the superficial nature of appropriating a label. The next level, appropriating surface features, addresses a similar surface-level knowledge of features without understanding how these features contribute to the overall meaning for the tool. Grossman et. al (1999) describe the next level, appropriating conceptual underpinnings, as understanding theoretical underpinning informing the use of the tool. Those that understand the theoretical basis for use of the tool can also use the tool in different situations. The last level, achieving mastery, links appropriation and mastery (effective use of the tool) and takes years of practice to accomplish.

Activity theory applied to this study for TC facilitation of MP3. Activity theory is a useful framework for understanding the process of learning to teach, particularly understanding how teacher preparation methods influence TCs' conceptions of teaching. Specifically, Engestrom's (1993) model provided a useful avenue to think about the complex relational work of facilitating MP3. Further, studying interactivity within the system can reveal any invisible influences not initially presumed to play a role in the system. Figure 2 illustrates the specific model of an activity system in teacher preparation used in this dissertation.

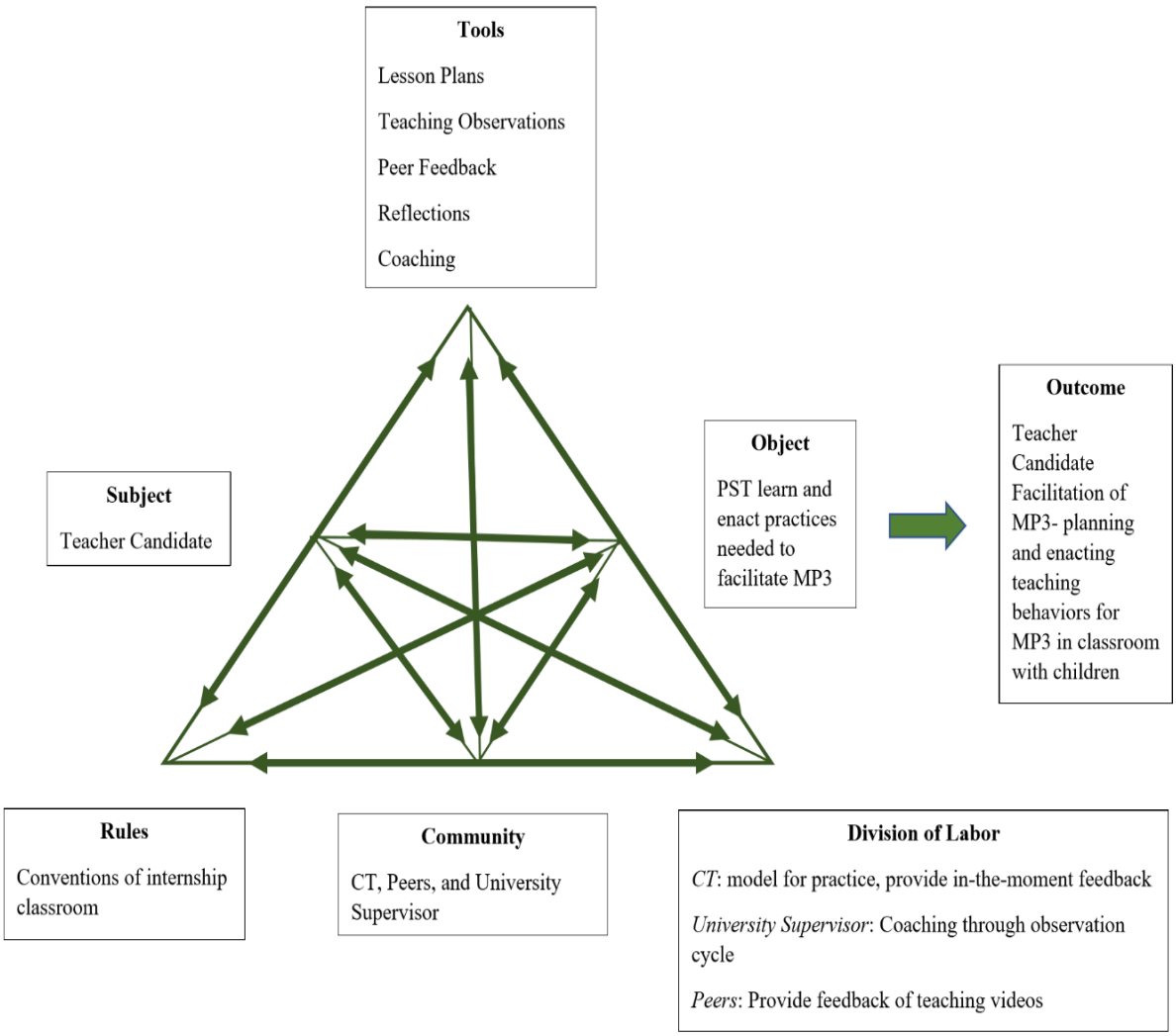

Figure 2. Activity System for Practice-Based Methods Course

Through the lens of activity theory, a practice-based methods course was considered a system for supporting TCs (subject) to learn the core practices needed to facilitate MP3(outcome) where collaborating teachers CTs and University Supervisors (community) coach TCs in enacting practices with students (rules) influenced how TCs learned to facilitate MP3. TCs wrote lesson plans, reflected on lessons, provided and received peer feedback of lessons (mediational physical tools) in learning how to facilitate MP3. Additionally, activity theory provided me a way to include how people in the activity system played a role in learning to teach, but also consider how they influenced their learning (division of labor).

Context. An activity system is grounded in an intentional contextual situation. This context gave the activity system its meaning and connected a series of otherwise random events and therefore played an essential role in the outcome. For this study, I intended to better understand practice-based methods contexts by studying the ways TCs facilitated MP3 in these settings. Further, I intended to learn how practice-based settings influenced TCs facilitation in order to make meaningful improvements to future math methods and clinical internship courses.

Tools. The tools intended for this study aimed to support TCs in planning and enacting teaching behaviors for MP3. The tools were intentionally used in and are associated with the practiced-based math methods courses and corresponding clinical internships for the purpose of supporting TCs facilitate MP3. The written lesson plan aided TC to explicitly plan teaching behaviors for MP3. Teaching observations were used as representations of teaching to help TCs develop a vision of teaching behaviors for engaging students in MP3. Reflections and coaching of teaching played a role in "unpacking teaching in ways that give students access to the pedagogical reasoning, uncertainties and dilemmas of practice that are inherent in understanding teaching as being problematic" (Loughran, 2007, p. 6). This is important for making visible the

teaching behaviors and their influence on further teaching practices involved in engaging students in MP3. More detailed descriptions of the written lesson plan (Appendix D), teaching observations, reflections, and peer feedback tools are included in chapter three, under the data sources section.

Division of labor. Division of labor or, those that share responsibility in TCs facilitation of MP3 included CT, University Supervisor/Math Methods Course Instructor, and Cohort Peers. It was initially indicated that CTs collaborate with TCs to plan teaching behaviors for engaging students in MP3 while also providing co-teaching techniques for supporting TCs enactment of teaching behaviors during the lesson. Supervisor/Math Methods Course Instructor would provide the most support in planning teaching behaviors as a result of pre-conference observation meetings. Peers initially were thought to share responsibility in the facilitation of MP3 by providing feedback on teaching behaviors and become a collaborative source for planning lessons.

Community. It was anticipated that CTs, Peers, University Supervisor/Math Methods Course Instructor played influential roles as members of the activity system. These individuals had the closest relationships with TCs across both university and clinical internship settings.

Rules. The rules of an activity system are those entities that guide, limit, dictate or regulate the ways action happens within the setting of the system. At the onset of the study, I expected conventions of the clinical internship classroom to be the most important regulating factor for TCs facilitation of MP3. I assumed conventions such as classroom norms and management strategies played a role in dictating how TCs facilitated MP3 in their clinical internship classrooms. This perspective indicated I had not anticipated the many complexities of learning to teach to become visible as a result of this study.

***Practiced-based methods of learning to teach*.** The activity system explored in this study, practice-based math methods course, was conceptually grounded in McDonald, Kazemi, & Kavanagh's (2013) Cycle for Collectively Learning to Engage in an Authentic and Ambitious Instructional Activity framework. The learning cycle was also intended to influence teacher educator's instruction of pedagogical practices for teacher education. The cycle illustrates the way in which the practice-based methods course (activity system) is constructed in this study (Figure 3) and how it supports TCs in learning particular practices "by introducing them to the practices as they come to life in meaningful units of instruction, preparing them to actually enact those practices, requiring them to enact the practices with real students in real classrooms, and then returning to their enactment through analysis" (p. 5). The context of the practiced-based based math methods courses are detailed further in chapter 3.

Summary. Activity theory is a useful framework for understanding the process of learning to teach, particularly understanding how teacher preparation methods influence TCs' conceptions of teaching. Through the lens of activity theory, a practice-based methods course can be considered a system for supporting TCs (subject) to learn the core practices needed to facilitate MP3(outcome) where collaborating teachers CTs and University Supervisors (community) coach TCs in enacting practices with students (rules) can influence how TCs learn to facilitate MP3. I intended to explore the complex settings and their interactions for which TCs facilitate Common Core MP3 within the activity system of a practice-based methods course. Exploration of this complex system allowed me to capture interactivity within the system and reveal invisible influences not initially presumed to play a role in the system.

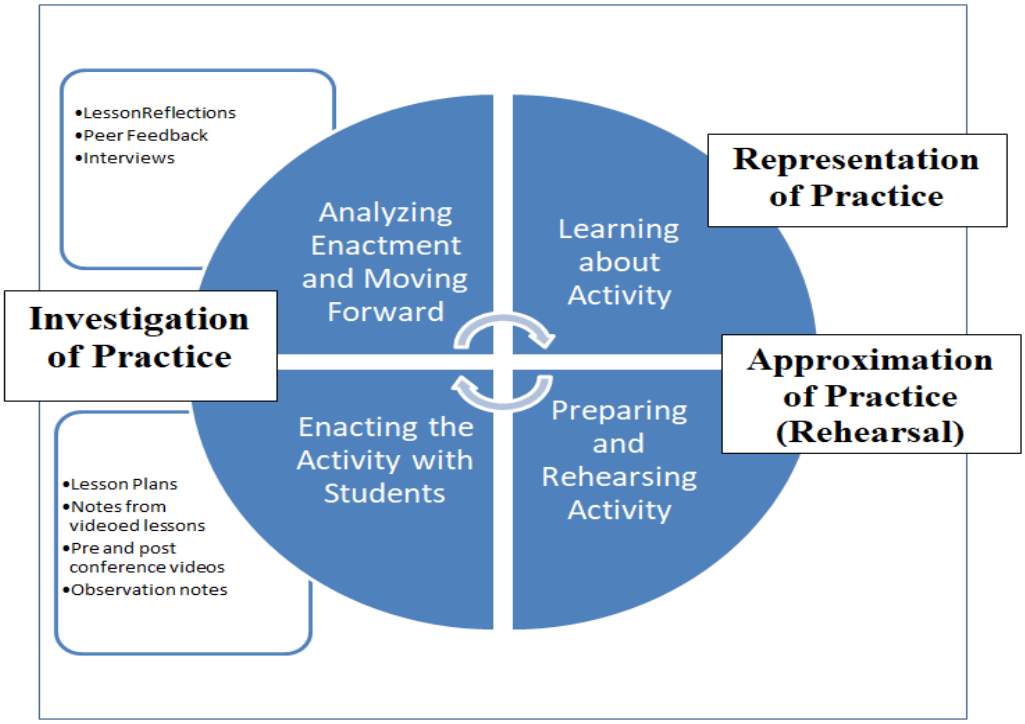

Figure 3. Practiced-Based Cycle for Learning to Facilitate MP3

I used a qualitative multiple case study to explore the ways TCs' learn to facilitate Common Core MP3 within the activity system of a practice-based methods course and accompanying clinical internship placement in an elementary classroom. To gain a better understanding of the ways TCs learn to facilitate MP3 across practice-based math methods and clinical internship classrooms, I explored the similarities and differences in the ways each TC planned and enacted teaching behaviors for MP3. The following research questions helped me understand how TCs learn to facilitate MP3.

Research Questions:

Within the context of a practice-based methods course and an accompanying clinical internship placement in an elementary classroom, how do three TCs learn to facilitate Common Core MP3?

- How does the TC plan to facilitate MP3 in their clinical internship classrooms?
- How does the TC enact teaching behaviors for facilitating MP3 in their clinical internship classrooms?
- In what ways do TCs perceive supports and tensions within the activity system of practice-based methods courses and clinical internship classrooms when facilitating MP3?

Definition of Terms

Teaching practice. Lampert (2004) defines *teaching practice* as beyond "what teachers do", but more specifically, the intentional behavior enacted with students (p.2).

Core teaching practices. Also referred to as high leverage teaching practices, these are intentional, in-the-moment, and high-frequency teacher behaviors, activities, and moves that depend on a teachers' professional knowledge. Grossman, Hammerness, and McDonald (2009, p. 277) define the characteristics of core practices as:

- Practices that occur with high frequency in teaching;
- Practices that novices can enact in classrooms across different curricula or instructional approaches;
- Practices that novices can actually begin to master;
- Practices that allow novices to learn more about students and about teaching;
- Practices that preserve the integrity and complexity of teaching; and
- Practices that are researched-based and have the potential to improve student achievement

Clinically based teacher preparation. Teacher preparation programs centered on supervised experiences in K-12 classrooms that support the development of "knowledge, skills, and dispositions" of practice (NCATE, 2010, p. 3)**.**

Practice-based methods. An approach to teacher education centered on learning, rehearsing, and enacting core practices for teaching.

Clinical internships. Experiences in K-12 classrooms that provide Teacher Candidates the opportunities to practice teaching.

Teacher candidate (TC). A student in a teacher preparation program.

University supervisor. A part/full-time university faculty hired to coach and evaluate Teacher Candidates in clinical internship experiences.

Collaborating teacher (CT). "P-12 school personnel and professional education faculty responsible for instruction, supervision, or assessment of candidates during clinical internship and clinical practice" (NCATE, 2010, p. 6)

Elementary mathematics methods course. A course in a teacher preparation program designed to teach mathematical content and math teacher pedagogical knowledge to Teacher Candidates.

Mathematical Practice 3. The Standards for Mathematical Practice located in the Common Core State Standards present eight practices that define how students become proficient in mathematics. These practices rest on researched proficiencies and methods developed by NCTM's process standards and strands for mathematical proficiency outlined in National Research Council's report (NRC) Adding It Up. Mathematical Practice 3, Constructing Viable Arguments and Critique the Reasoning of Others, states that students rationalize their answers, communicate them to others, and respond to others' reasoning and arguments.

Proving events. Those situations when students share justifications for their reasoning and provide evidence for their claims. In other words, elementary school children should provide justification for why their claims (reasoning, arguments, or problem solutions) are valid or invalid (true or not true). This definition denotes an argument by determining the validity of a statement or idea using symbolic (visual representations such as tape diagrams or number bonds), non-symbolic (manipulatives), and verbal (explanations) representations (Stylianides & Stylianides, 2014).

Chapter Two: Literature Review

Learning to Teach Mathematics

Now more than ever, our nation faces a demanding task to fill K-12 classrooms with knowledgeable, prepared, and competent teachers, including novice teachers. The knowledge teachers need to be successful in the classroom is often invisible to untrained observers (Darling-Hammond, 2006). Schulman (1987) posited the knowledge needed for effective teaching should be abundant in both knowledge of content *and* pedagogy for teaching. Every day teachers need to make complex, in-the-moment judgment decisions based their knowledge for teaching.

Building on to Schulman's ideas, Ball, Thames, and Phelps (2008) further define the complex domains of mathematical knowledge needed for teaching as more than knowing subject matter content and pedagogy, but also knowledge of curriculum, students, and teaching. Knowledge of Content and Students (KCS) combines knowing about students and specialized content knowledge, while Knowledge of Content and Teaching (KCT) merges knowing about teaching and knowing about mathematics. In other words, teachers should anticipate student thinking, be familiar with common misconceptions and conceptions, be able to hear and interpret student thinking, and make instructional decisions based on specific knowledge of math and pedagogical issues affecting students. Thus, learning the complex relational work attending to teaching mathematics requires teacher preparation curricula to be grounded in learning to enact core instructional practices (Ball, Thames, & Phelps, 2008; Ball & Forzani, 2009; Grossman, Hammerness, &McDonald, 2009; Lampert, 2010; Zeichner, 2012; Forzani, 2014).

Elementary Mathematics Teacher Preparation Focused on Core Practices

Ball and Cohen (1999) defined learning practice to be "centered in the critical activities of the profession" (p. 13) through providing opportunities for learning by, first, identifying practices and then, followed by choosing and constructing resources representing those practices. Additionally, Ball and Cohen (1999) suggest TCs engage in investigations of practice to question, investigate, analyze, and critique in preparation to use professional knowledge in practice.

Ball and Forzani (2009) further define practice-based curriculum as the "unpacking and specifying practice in detail and designing professional education will offer novices multiple opportunities to practice the work and to fine-tune their skills" (p.498). Ball and Forzani (2009) explain teaching requires a set of specialized skills and complex practices (high leverage teaching practices) that one does not learn naturally and invisible to the untrained observer, such as identifying a variety of ways students think about a math problem, probe ideas, or think about why and how a student arrived at an incorrect answer.

Practice-based teacher education involves TCs learning, enacting, and rehearsing specialized pedagogical or instructional practices for teaching (McDonald, Kazemi, & Kavanagh, 2013). Many researchers are attempting to identify the complex practices needed for teaching (e.g. Kazemi, Franke, & Lampert, 2009; Ball & Forzani, 2009; McDonald, Kazemi, & Kavanaugh, 2013; Bailey & Taylor, 2015). Others aim to explore the ways they define their practice around how core practices are broken down and made visible to TCs (e.g. Kazemi, Ghousseini, Cunard, & Turrou, 2016; Peercy, & Troyan, 2017). Teacher educators have found engaging TCs in mediated clinical internships (Campbell & Dunleavy, 2016); deliberately created practice opportunities (Ghousseini & Herbst, 2016); reflection of experiences (Bailey & Taylor, 2015),

and rehearsal (Anthony, Hunter, & Hunter, 2015; Ghousseini, 2017) can support learning and development of core teaching practices.

Grossman, Hammerness, and McDonald (2009) acknowledge definitions of core practices differ across researchers however, they share the same characteristics. Grossman, et al. (2009) define those characteristics as practices that have high-frequency occurrence in teaching, can be implemented by TCs despite varying curriculum and instructional methods; allow TCs to learn about teaching and learning, uphold the complexity and integrity of teaching, are grounded in research, and have the ability to improve student learning.

Core Practices for Teaching

Researchers are working to identify core practices for teaching elementary mathematics (e.g. Kazemi, Franke, & Lampert, 2009; Ball & Forzani, 2009; McDonald, Kazemi, & Kavanaugh, 2013). Such as facilitating classroom discussion (Ghousseini, 2015; Ghousseini & Herbst, 2016), attending to students mathematical thinking (Anthony, Hunter, & Hunter, 2015); and embedding instructional activities presenting opportunities for TCs to enact and rehearse core practices such as choral counting (Kazemi, Ghousseini, Cunard, & Turrou, 2016; Lampert et al., 2013).

Researchers such as, Grossman, et al (2009), have begun to reimagine the curriculum and assert teacher educators, first "need to compose the practice of teaching into its constituent parts in order to identify the core practices that could then provide the basis for such a curriculum" (p. 278). Grossman, et al (2009) named the following as overarching core practices (many practices and routines compose each of these); establishing a classroom culture, learning about student understanding, and facilitating classroom discussions. For example, learning about student understanding includes eliciting student thinking, anticipating student responses, and eliciting

further thinking (Anthony, Hunter, & Hunter, 2015; Ghousseini, 2015; Ghousseini & Herbst, 2016).

Teacher Educator Pedagogies of Enactment

The shift towards a practice focused stance of teacher preparation requires interactive methods for the development of teaching theories and practices, which places considerable demands on teacher educators (Grossman, et al., 2009). As a result, teacher educators are exploring the ways they define their practice around these the core practices, and share the work being done with elementary TCs (e.g. Peercy, & Troyan, 2017; Kazemi, Ghousseini, Cunard, & Turrou, 2016). Teacher educators have found engaging TCs in approximations, decompositions, representations of practices(Ghousseini, 2015; Ghousseini & Herbst, 2016); mediated clinical internships (Campbell & Dunleavy, 2016); deliberately created practice opportunities (Ghousseini & Herbst, 2016); reflection of experiences (Bailey & Taylor, 2015), and rehearsal (Ghousseini, 2017; Anthony, Hunter, & Hunter, 2015) can support learning and development of core teaching practices.

Ghousseini (2015) and Ghousseini and Herbst (2016) describe the complexity of facilitating classroom discussion because of the many intricate skills and routines needed to carry out productive and meaningful collaboration. For example, Ghousseini (2015) described collaboration among students to be an integral component for math discussions which relies on building relationships with and between students that require skills for listening and understanding to others' ideas for the purpose of learning mathematical content. Inclusive in those ideas, teachers need to know how to elicit, assess, and respond to students' thinking, and while simultaneously using a variety of representations to support students in communicating

their thinking to others (Ghousseini, 2015). In learning these complex skills, TCs may only understand problems of practice once they get into authentic teaching situations.

Approximations, decompositions, and representations of practice. Ghousseini (2015) employed a case study methodology to delve into a deeper understanding of problems of practice associated with practice-based teacher preparation. In a secondary math methods course structured around the rehearsal of practices for teaching mathematics which Ghousseini (2015) decomposed into three smaller practices comprising of establishing norms, explaining concepts of mathematics, and leading a class discussion in mathematics. Each of these included supporting nested skills important to carrying them out consisting of eliciting students' thinking, representing students' thoughts, and focusing on important mathematical components of students' responses. Math teacher educators intentionally modeled the practices, followed by decomposition where teacher educators broke down, labeled and highlighted practices (and discussed problems of practice that might arise when enacting it in classrooms with students).

After decomposition, three opportunities to enact the practices through the approximation of teaching included two occasions in-class where TCs constructed dialogue using a script and fishbowl set up, and once leading classroom discussion at their field placements. The study helped the researchers understand some problems of practice TCs may face when enacting core practices. Including adapting instruction and making decisions based on pedagogical and content knowledge to address the continuously changing needs of the classroom environment. The researchers also confirmed the perception that teaching is complex is due to the in-the-moment responsiveness necessary for attending to students' thinking.

In a very similar study, Ghousseini and Herbst (2016) used a qualitative study to investigate pedagogies supporting TCs' learning to implement core practices for teaching. Similarly, in this

study, I used approximation, decomposition, and representation of practice as a way to attend to practice-based teacher education. Teacher educators intentionally helped TCs learn instructional moves used in class discussions. The teacher educators modeled some of the instructional moves as representations of practice. After, the teacher educators moved onto decomposition of the instructional moves which involved TCs naming, breaking down, and elaborating on representations of practice. Approximation of practice followed which comprised of enacting the representations of practice in clinical internships and methods class using instructional methods of a fishbowl and constructed dialogue. The engagement in all three pedagogies of practice are powerful when they were intentional and varied to provide a balanced approach allowing TCs to learn about, acquire skills and enact them.

Grossman, et al (2009) define approximations of practice to "include opportunities to rehearse and enact discrete components of complex practice in settings of reduced complexity" (p. 283). Further, Lampert, et al. (2013) describe three characteristics of rehearsals to include (1) occurrences around shared instructional activities, (2) to be interactive, and (3) teacher educator playing a central role for feedback and support. Rehearsals can be a vehicle for learning how to respond to students' thinking (Anthony, Hunter, & Hunter, 2015) and learn mathematical knowledge for teaching (Ghousseini, 2017). It is also suggested rehearsals of ambitious or core teaching practices may lead to a greater understanding of current teaching practices despite novice teachers' experiences learning mathematics differently (Bailey & Taylor, 2015).

Anthony, Hunter, and Hunter (2015) aimed to explore how professional noticing within a succession of rehearsal opportunities in a math methods course and classroom setting lends itself to learn how to listen and respond to students' thinking. In this study, TCs rehearsal of teaching practices were video-recorded and were coached in-the-moment by a mathematics teacher

educator. The researchers noticed coaching and rehearsing activities supported TCs in learning about how to respond to students' mathematical thinking.

Ghousseini (2017) studied how rehearsals could support TCs in a master's level elementary teacher certification program learn mathematical knowledge for teaching (MKT) (knowledge, skills, habits of mind needed to be able to teach mathematics). This study employed instructional activities including choral counting and strategy sharing. Ghousseini (2017) found rehearsals when embedded and deliberate in math methods courses were beneficial for providing opportunities for TCs to practice MKT.

Bailey and Taylor (2015) explored the ways novice teachers (graduate students in a 12-week elementary mathematics methods course) considered and reflected on problem-solving teaching activities involving ambitious teaching practices. The researchers found a lack of prior experiences with teaching with problem-solving, and opportunities to practice ambitious teaching practices led to a shift in thinking about how the novice teachers support children's learning.

Instructional activities. Instructional activities (IAs), such as choral counting, quick images, strings, or strategy sharing, were executed in classrooms and provided novice teachers with structured support from teacher educators (Kazemi, Franke, and Lampert, 2009). They serve as containers for rehearsing and discussing aspects of ambitious teaching practices (Kazemi & Waege, 2015). IAs can be used across environments including math methods courses and clinical internships (Anthony, Hunter, & Hunter, 2015; Kazemi & Wage, 2015)

Within their methods class, Kazemi, Franke, and Lampert (2009) prepare novice teachers for ambitious practice using carefully selected instructional activities where opportunities to analyze and critique pedagogy support teacher learning. Kazemi and Waege (2015) and Anthony,

Hunter, & Hunter (2015) explored the ways instructional activities, such as choral counting, quick images, strings, were implemented by TCs in two settings, rehearsals in math methods courses and in small group instruction in a school-based classroom. Kazemi and Waege (2015) found few TCs pursued questioning practices encouraged students to create justifications for their mathematical ideas, suggesting complex practices may demand a deeper understanding and more opportunities for rehearsing it.

Teacher Learning within Practice-Based Math Methods Courses

Approximation of practice using rehearsals. Four studies attended to TCs learning using rehearsals of practice within math methods courses. Through rehearsals, TCs were able to practice in-the-moment instructional decision making with eliciting student thinking or ideas as occurring the most often (Lampert, et al., 2013; Kazemi, et al., 2016; Ghousseini, 2017). Three of these studies (Lampert, et al., 2013; Kazemi, et al., 2016; Ghousseini, 2017) took place in a master's level elementary math methods course.

Consecutive studies (Lampert, et al. 2013; Kazemi, et al. 2016) used a cycle of enactment and investigation which allows TCs to "travel back and forth between methods course to enactment in schools" (Lampert, et al., 2013, p. 228). From their findings, they concluded rehearsals provide rich opportunities for master level TCs in elementary methods course to learn the broad scope of complex requisites for teaching. They also discovered through their study the ways rehearsals created opportunities for TCs to experience a variety of teaching practices and to make sense of complex interactions taking place between students and teachers.

Similar to the previous two studies, Ghousseini (2017) provided opportunities in a recurring cycle of enactment and investigation pivoting on IAs to support TCs in the development of knowledge about teaching and mathematics and interactions with children. Implied from the

study is the possibility for methods course to be places for TCs to develop pedagogical and content knowledge through the use of rehearsals. Ghousseini found evidence TCs did elicit and respond to students during rehearsal events, however, they lacked evidence the TCs learned mathematical knowledge needed for teaching. Ghousseini (2017, calls for more research focusing on how this knowledge demonstrated in rehearsals materializes in classroom teaching.

Anthony, Hunter, and Hunter (2015) also used IAs within rehearsal cycles of enactment and investigation focused on TCs learning of professional noticing. They found early rehearsals revealed TCs were able to elicit student thinking, but would quickly move on without responding to or exploring connections among students' solutions. In addition, TCs learned active listening was important for students when sharing explanations. TCs learned they needed to provide and facilitate opportunities for students to share and hear others' ideas and establish routines and norms for this collaborative participation. Later, TCs went from having peers engage with a particular response to using response as a resource to extend discussion. TC learning exhibited a trajectory "initially focused on eliciting, then responding, and then building on and connecting students' thinking to the mathematical goals of the lessons" (Anthony, Hunter, & Hunter, 2015, p. 20).

Representation, approximation, and investigation of practice. Two studies attended to practice-based math methods courses involving representation, approximation, and investigation of practice (Kazemi & Waege, 2015; Ghousseini & Herbst, 2016). In a graduate-level elementary math methods course, TCs began their understanding of eliciting and responding to students' thinking by encouraging students to explain how they solved a math problem. Later, only one TC (out of three) engaged in asking questions to help students develop mathematical justifications. Kazemi and Waege (2015) may suggest this type of instructional move is more complex and

requires further instructional attention by teacher educators to develop mathematical conversations. When representing students' thinking, TCs wrote it up on the board and did not relate different representations to each other.

In a secondary math methods course, a teacher educator sought to develop TC understanding the practices and instructional moves related to mathematical discussions including conjecturing, justifying, and critiquing through representation, approximation, and investigation of practice events. Ghousseini and Herbst (2014) concluded these events together were powerful in providing TCs to decompose, unpack, and experience learning pedagogies of practice. As a result, TCs exhibited evidence of increased content knowledge, eliciting and responding to student thinking, and a repertoire of instructional moves for facilitating mathematical discussion, encouraging students to make conjectures, and justifications.

In these studies, researchers confirmed practice-based experiences supported TCs to develop knowledge and understanding the core practices of classroom discussion focused on students' thinking. Four out of six studies took place in graduate-level elementary math methods courses, one took place in an undergraduate elementary math methods course, and one took place in a secondary education math methods course. We cannot assume the learning happening in these courses would be different. However, we can predict similarities among studies where learning to teach is occurring.

Practices and Pedagogies for Teaching Mathematics Practice 3

The Common Core State Standards (CCSS) for Mathematical Practices (2010) are research-proven praxes resting on the National Council for Teachers of Mathematics' (NCTM) (2000) set of core mathematical teaching practices and National Research Council's (2001) strands for mathematical proficiency. NCTM's teaching practices describe the essential teaching skills

derived from the research-based learning principles, as well as other knowledge of mathematics teaching has emerged over the last two decades. NRC's report, Adding It Up described the strands for mathematical proficiency as a "framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency" (p.116). CCSS MP3, construct viable arguments and critique the reasoning of others, states students "proficient in mathematics should be able to justify their conclusions, communicate them to others, and respond to arguments of others" (2010, p. 117). For teachers, this means they need to know how to engage their students in rich meaningful discourse around justification and includes, encouraging responses to other students and using the ideas of others as a source to deepen understanding of mathematical procedures and concepts. According to Bostic, Matney, and Sondergeld (2017) teachers facilitating MP3 should,

- Provide and orchestrate opportunities for students to listen to the solution strategies of others, discuss alternative strategies or solution(s), and defend their ideas,
- Ask higher-order questions which encourage students to defend their ideas, consider student(s) response(s) before making code,
- Provide prompts/tasks that encourage students to think critically about the mathematics they are learning, must be related to argumentation or proving events, and,
- Engage students in proving events that encourage students to develop and refine mathematical arguments (including conjectures) or proofs (p. 8).
- Developing this type of individual and collaborative work with students requires substantial skill on behalf of teachers to develop mathematical understanding such as, facilitating class discussion, supporting student participation, and providing feedback to students (Ghousseini, 2015).

Proving, Reasoning, and Justification

Engaging students of all ages, including elementary children, in proving events allows for a deeper understanding of the mathematics (Stylianides, 2016). However, there is limited attention focused on students in elementary school engaging in proving events (Stylianides, 2016). Lack of proving in the elementary grades can be attributed to associating proving with geometrical proofs and limited understanding of what it means to engage in proving events at these grade levels (Stylianides, Stylianides, & Schilling-Traina, 2013). Currently, there are different perspectives for the definition of proof versus engaging in proving events. It is important here to address the difference between them because MP3 engages students in proving events. Andreas Stylianides and his colleagues (2007a; 2007b; 2008; 2016) have worked to develop what proving events look like with elementary children and to define the role teachers play in fostering justification with students. For the purpose of this study, I align with Stylianides'(2007b) definition for the meaning of proof:

Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:

1. It uses statements accepted by the classroom community (set of accepted statements) that are true and available without further justification;
2. It employs forms of reasoning (modes of argumentation) that are valid and known to, or within the conceptual reach of, the classroom community; and
3. It is communicated with forms of expression (modes of argument representation) that are appropriate and known to, or within the conceptual reach of, the classroom community. (p. 291)

In other words, elementary school children should provide justification for why their claims (reasoning, arguments, or problem solutions) are valid or invalid (true or not true). This definition denotes an empirical argument by determining the validity of a statement or idea using symbolic (visual representations such as tape diagrams or number bonds), non-symbolic (manipulatives), and verbal (explanations) representations. This is not to be confused with a proof, as a proof shows how a claim or generalization is true for all cases. (Stylianides & Stylianides, 2014). In elementary classrooms, justification is often referred to when students communicate explanations to convince others that their reasoning, or solutions, are correct. MP3 further suggests that, in turn, students listen to others' justifications to decide if their reason is valid (or invalid). Proof pushes justification further by supporting arguments with evidence, such as modes representations, so others may accept the claim as well (Bostic, 2016). Those situations when students share justifications and provide evidence for claims are considered *proving events* and are addressed in MP3.

Research suggests there is insufficient preparation of elementary TCs to engage students in proving events in mathematics classrooms (Ball & Stylianides, 2008). Engaging students in reasoning and proving events is challenging for a couple of reasons. First, teachers, particularly elementary teachers, may lack understanding about reasoning and proof (Goulding, Rowland, & Barber, 2002; Morris 2002). Second, teachers may believe engaging in proving events around justification and reasoning is outside the ability level of their students (Knuth, 2002). However, there is little evidence to provide a clear understanding of what other barriers may hinder teachers enacting instruction supporting proving events in their classrooms (Stylianides, 2013).

Providing Opportunities for Students to Engage with Other's Mathematical Ideas

I would be remiss if I didn't address the role discussion plays in facilitating MP3. Orchestrating and facilitating discussion have been proven to be a core practice for teaching because of its dependence on knowledge of student learning, in-the-moment decision making, and being responsive to students' contributions (Kazemi et al 2009; Stylianides & Stylianides, 2014). Supporting students to communicate their ideas with others and engage with others' ideas are grounded in knowing how to orchestrate opportunities for students to have discussions around specific subject matter content. Teachers who successfully facilitate MP3 provide opportunities for talking and listening between teacher-student and between student-student (Bostic, Matney, & Sondergeld, 2017).

First teachers must encourage students to discuss or explain their reasoning. Franke, Turrou, and Webb (2015) found that invitation and support moves supported teachers in providing opportunities for students to listen and discuss another's mathematical ideas. Teachers in the study prompted engagement by using invitation moves and followed with support moves which were in-the-moment reactions to students, probing thinking, scaffolding, and positioning to foster these discussions. In addition to prompting students to discuss their reasoning, listening to student responses plays an important role in deciding whether students are making sense of the mathematics at hand (Mueller, Yankelewitz, & Maher, 2014).

MP3 also requires teachers to provide opportunities for students to engage in mathematical discussions beyond just "show and tell" talk to develop deeper mathematical knowledge (Stein, Engle, Smith, & Hughes, 2008). To support teachers in such productive discussion, Stein et al. (2008) posited they should make connections between various student responses by comparing

and contrasting solutions, highlighting errors and more efficient solution methods, and allowing students to reflect and refine their own solutions.

Summary

These studies have provided evidence of the benefits of practice-based teacher preparation for supporting teachers to learn core practices for teaching mathematics. Additionally, it has been suggested by these studies when instructional activities and opportunities for rehearsal are exercised in an integrated fashion (Grossman et al., 2015) connected to clinical internships or authentic settings (Campbell & Dunleavy, 2016; Anthony, Hunter, & Hunter, 2015; Kazemi & Wage, 2015), involve varied, balanced and intentional pedagogies of enactment such as approximation, decomposition, and representation of practices (Anthony, Hunter, & Hunter, 2015; Ghousseini, 2015; Ghousseini, 2017; Ghousseini & Herbst, 2016), and provide time for acquiring knowledge, rehearsal, and reflection of practices (Ghousseini, 2015; Ghousseini, 2017; Ghousseini & Herbst, 2015) they can be significant learning experiences of core teaching practices for TCs.

The shift towards these pedagogies requires skilled coaching and immediate feedback on the part of teacher educators (Grossman, et al, 2009). Coaching should also include supporting preservice teacher reflection of challenges and how to address them. It is also suggested when TCs work in authentic situations in P-12 classrooms, it may require math methods instructors to also be supervisors to access and coach what is happening in the field as well as make connections from theory to practice and practice to theory (Ghousseini, 2015).

The studies above answered the call for further research grounded in practice-based teacher education in which to identify; successful research-based core practices, successful designs of courses for methods classes, and learning trajectories of TCs' leaning (and challenges) of these

practices. When we are supporting the learning of complex practices of teaching, our work as teacher educators "must be informed by what novice teachers do and find challenging in practice" (Ghousseini, 2015, p. 335).

Further, many of these studies were conducted in secondary math methods courses or master's level methods courses, and work done with elementary TCs can look different. For example, specialized content knowledge, knowledge of elementary student learning, and ways to facilitate conversations about math required in elementary grades are quite different than secondary education. In this study, I respond to the calls to better understand the ways elementary TCs begin to learn and enact these practices in hopes of informing the ways teacher educators can support their learning. In addition, this study will add to the practice-based teacher preparation conversation regarding learning how to teach mathematics.

Chapter Three: Methodology

I used an exploratory descriptive multiple case study to explore the ways TCs learn to facilitate Common Core MP3, "construct viable arguments and critique the reasoning of others" (CCSSM, 2010), within the activity system of a practice-based methods course and accompanying clinical internship in an elementary classroom. The purpose of an exploratory descriptive multiple case study was to gain a better understanding of a *quintain* through the examination of its cases. According to Stake (2006), a *quintain*, or the phenomenon being studied, is made up of a collection of cases each having their own "problems and relationships" (Stake, 2006, p. vi). Stake (2006) asserted, "in multiple case study research, the single case is of interest because it belongs to a particular collection of cases (p. 4). An exploratory descriptive multiple case study allowed me to look at how each case operates within the quintain and explore the "experiences of real cases operating in real situations" (Stake 2006, p. 3). To gain a better understanding of the ways TCs learn to facilitate MP3 across a practice-based math methods and clinical internship classrooms, I explored the similarities and differences in the ways each TC planned and enacted the facilitation of MP3.

The exploratory descriptive multiple case study allowed me to "find out firsthand what each individual case does" (Stake, 2006, p. 27) and then, how TCs' experienced learning to facilitate MP3 compared to each other. I used the components of activity theory (object, subject, tools, community, division of labor, and rules), as the lens through which I analyzed TCs' learning from the perspective of those involved by "appreciating the uniqueness and complexity of the case, its embeddedness, and interaction with its contents" (Stake, 1995, p. 16).

Additionally, Ghousseini (2015) argued that this type of investigation into TCs' experiences learning complex mathematical teaching practices could reveal challenges and struggles that are important for teacher educators to understand. The following research questions guided me as I came to understand how TCs facilitated MP3.

Research Questions:

Within the context of a practice-based methods course and an accompanying clinical internship in an elementary classroom, how do three TCs facilitate Common Core MP3?

- How do TCs plan for the facilitation of MP3 with students in their clinical internship classrooms?
- How do they enact teaching behaviors to engage students in MP3 within their clinical internship classrooms?
- In what ways do TCs perceive supports or tensions within the activity system of practice-based methods courses and clinical internship classrooms when facilitating MP3?

Case Study Design

I want to better understand the ways that TCs learn to engage in high-leverage practices for teaching math, specifically how they learn to facilitate MP3 in which students construct arguments and evaluate the reasonableness of others' arguments. The research design was an exploratory descriptive multiple case study exploring the experiences of TCs learning to facilitate the mathematical practices throughout their practice-based math methods course and simultaneous clinical internship in elementary classrooms. Stake (2006) asserts single cases are not representative of other cases, however, interactions between cases may be relevant in making meaning of the quintain being studied. Miles, Huberman, and Saldana (2014) state,

> multiple-case sampling adds confidence to findings. By looking at a range of similar and contrasting cases, we can understand a single-case finding, grounding it by specifying how and where and, if possible, why it carries on as it does. We can strengthen the precision, validity, stability, and trustworthiness of the findings (p.33).

In seeking an accurate understanding of how TCs learn to teach, a multiple case study design was appropriate because it allowed me to consider each TC's experiences in learning and enacting practices for teaching mathematics while also having a collection of experiences to compare (Stake, 2005).

Miles, Huberman, and Saldana (2014) acknowledge the differences between tight versus loose study designs. It is important my study was both tight enough to recognize teacher behaviors for facilitating MP3 with TCs, but loose enough allowing for alternative behaviors to emerge (Miles, Huberman, & Saldana, 2014). Further, too much focus on initial research contexts or questions can "distract researchers from recognizing new issues when they emerge" (Stake, 2006, p. 13). The literature does not provide an exhaustive or comprehensive collection of teaching behaviors or practices associated with MP3, and it was the aim of this study to explore the teaching behaviors of TCs. For this study, I used Bostic, Matney, and Sondergeld's (2017) validated tool as a guide for recognizing teaching behaviors associated with facilitating MP3. However, these Look-Fors are typically used with in-service teachers and may not be inclusive of all teaching behaviors enacted for MP3, particularly at the elementary school level. Further, I understood TCs may or may not have enacted these teaching behaviors and could have exhibited other behaviors that I explored more deeply. My design applied an in-between loose versus tight structure utilizing a looser initial data analysis allowing for examination of all

behaviors and experiences associated with MP3 to emerge while utilizing guiding questions for cross-case comparison.

I collected data from participants generated during their Math Methods I and II courses and clinical internship classrooms. I used the components of activity theory to gain a deeper understanding of how TCs planned and enacted facilitation MP3 in clinical internship experiences. I collected a variety of naturally occurring assignments from the methods course including; TCs lesson plans; notes from videos of TCs facilitating MP3 with students in clinical internship classrooms, and TCs' reflections of these lessons. Additionally, I conducted semi-structured interviews with TCs, after grades for the course were submitted, to confirm and strengthen interpretations of how TCs perceive the components of the activity system had influenced their pedagogical learning.

The Role of the Researcher

I have spent the last 20 years in the field of education including a classroom teacher and the Director of Education at a Children's Museum. While working in an urban school district, I was involved in STEM Grants and eventually became a District Math Leader and Coach. This position allowed me to engage in extensive training in the field of mathematics education and provide professional development seminars. Additionally, I was on the district mathematics curriculum committee in which I worked with a team to develop supplemental and assessment curriculum materials. I am a teacher at heart, although I am no longer in a classroom with children, I will always process my experiences through the lens of a teacher. Just like a child will take apart their favorite toy to see how it works, I too am curious about what makes great teachers. Teaching is complex which is causing a decrease in longevity within the teaching profession. I am passionate about how teacher educators can better understand how to unpack the

layers of teaching and better prepare TCs to enter and remain in the classroom. Through my past experiences in professional development and working with TCs in the teacher preparation program, I believe learning how to teach is grounded in Clinically based experiences embedded in clinical internship classrooms.

For this study, I was the researcher, the TCs' Math Methods II course instructor, and the university supervisor for the TC's clinical internships. I did not consider my roles as Math Methods Course Instructor and University Supervisor separate, rather for me they were intertwined as one teacher educator and aimed to support TCs consistently across all spaces of the program. My beliefs as a teacher educator are wholly based on the Constructivists theories of Dewey (1933), Piaget (1926) and Vygotsky (1996/2012), which pronounce learning is directed by and among students through experiences and activities in teaching. I believe one of the most important tasks of a teacher educator is to help TCs to unpack the foundational knowledge needed to be self-directed learners which is essential to perform teaching practices. I also believe good teaching requires consistent and continual reflection as well as a study into one's own practice. According to Loughran (2014), teaching and learning are dominated by the journey along an individual path whereby development and, growth is dependent on what that person sees and understands as important to his or her practice at that time. My role as a teacher educator in this study granted me deep and long access to TC's learning over their time in the teacher preparation program and allowed me to engage in this study as a participant-observer. My time spent with the TCs and CTs at clinical internship schools allowed me to build important meaningful and trusting relationships and deep insight and understanding of the context of this study. As no other people held the same role, this research study honored and captured my unique perspective from inside the activity system. Therefore, I intentionally used myself as a

tool for data collection to develop my knowledge and construct understandings into teaching and learning (Ball, 2000). Ball stated,

> studying teaching from the first-person perspective offers a special kind of personal inside view that is difficult to gain through even close participation observation. Because teaching and learning are deeply personal-that is, they are in fundamental ways relational and about persons-approaches to scholarship that use the personal as a resource offer the possibility of insights that are more difficult to gain from an outsider's perspective. Some aspects of experience are inarticulable, the chasm between what we know and what we can say is variable, but rarely closes (p. 392).

My first-hand knowledge of the happenings of both the Math Methods II course and clinical internship classrooms allowed access to a 360-degree view from inside the activity system to the contexts of this study and the interactions between them to better understand the meanings participants made within the activity system. Although I am situated within this study and played a role in this research, it is not considered a self-study. I was interested in exploring TC's experiences and their facilitation of MP3 as the unit of analysis rather than influences on my own practice as a math methods course instructor or university supervisor.

I acknowledge my authority over TCs in this study because I was both their supervisor and math methods course instructor. Therefore, participants for this study were not chosen until after final grades for the courses were submitted. According to Wong (1995) when a researcher is also in the role of instructor, the teaching should take precedence over the researching. I ensured that if any tensions arose between being a researcher and course instructor, I made choices that valued my role as an instructor over my role as a researcher. For example, when conducting a pre-conference with Vanessa, a participant in this study, she was struggling to understand the

math content in the first-grade lesson. I made the decision to spend our pre-conference meeting to support Vanessa in understanding math concepts and set aside my agenda for discussing the ways she planned to address MP3 within the lesson.

University Elementary Education Teacher Preparation Program Context

I am an elementary math methods course instructor and university supervisor at a large university in the southeast United States. The state-approved undergraduate elementary teacher education program includes coursework and extensive experiences in elementary school settings for intentional connections from theory to practice over five semesters. The program is also grounded in preparing teachers for a diversity of learners through the understanding of 4Is- inquiry, inclusion, innovation, and integrated instruction. Throughout the clinically- rich program, TCs spend over 1000 hours engaging in clinical internships with knowledgeable mentor teachers, or collaborating teachers (CTs), and university supervisors. With the help of their supervisors, TCs engage in systematic research to improve their teaching practice (inquiry) and have intentional opportunities for exploring, integrating, and teaching in core content subject areas (integration). TCs are provided experiences in innovative teaching including using instructional technology in their clinical internship classrooms (innovation). TCs earn an endorsement in English for Speakers of Other Languages (ESOL) by taking nine credit hours of ESOL coursework. All required coursework addresses equitable teaching practices including extensive study in differentiating instruction.

TCs enter the program after completing prerequisite education courses and passing a general knowledge examination; this typically is at the beginning of their junior year and they are enrolled in Level 1 clinical internship. During level I clinical internship, TCs spend one day a week over the duration of the semester (15 weeks) with an assigned collaborating teacher (CT)

and grade level at a local elementary school. TCs continue onto their second semester of junior year in Level II clinical internship, typically staying with the same CT and grade level. During the first semester of their senior year, TCs are in level three clinical internship and are assigned a different grade level and CT within the same clinical internship school. Typically, TCs are required to spend time in both intermediate and primary classroom, therefore transfers to an intermediate or a primary classroom to fill this requirement are made. TCs spend two days a week level three clinical internship classrooms. During the summer between level one and level three clinical internship, TCs complete an alternative clinical internship at local after-school programs. In the last semester of TCs' senior year, they completed final clinical internship where they typically stay with their level three CT and spent five days a week in their clinical internship classrooms.

The elementary teacher preparation program uses a cohort model with approximately 10 cohorts of TCs. In each cohort, approximately 30 TCs are placed across two partnership elementary schools. Each partnership school has approximately 15 TCs and is assigned to one university supervisor who has responsibility for supervising the clinical internships of the TCs. Each cohort stays together throughout their program and shares the same class schedules.

The supervisor typically remains with the cohort in all four levels of clinical internship from their entrance into the education program (junior year) until their graduation and plays a major role in helping the TCs to make connections between coursework and clinical internship. Each semester, university supervisors are responsible for holding weekly seminars with TCs, engaging TCs in two observation cycles (pre-conference, observation, post-conference), supporting the TCs inquiry process, and maintaining partnerships with clinical internship school administration,

CTs, and staff. In order to do this, supervisors spend quite a bit of time at their clinical internship schools, typically eight or more hours a week.

The elementary education program of study required TCs to take two math methods courses; Mathematics Methods I was taken during Level II clinical internship (spring semester of their junior year), and Mathematics Methods II was taken during Level III clinical internship (fall semester of their senior year). Table 1, Overview of Teacher Candidate Clinical internship and Math Methods Course Sequence, illustrate the correspondence between clinical internships and math methods courses. This study spanned a year and a half of TCs time in a teacher preparation program and took place across both math methods courses and continued into final clinical internship. The methods course meets once a week for two hours and 45 minutes, and TCs are in their clinical internship classrooms for two full days per week for the duration of the semester (15 weeks).

Table 1
Overview of TCs' Clinical Internship and Math Methods Course Sequence

Year in the Teacher Preparation program	Semester	Clinical internship Level	Days spent in clinical internship classroom	Corresponding Math Methods Course
Junior	Fall	Level 1	1 day/week	
	Spring	Level 2	1 day/week	Math Methods I
	Summer	Alternative	4 days/week (3 weeks)	
Senior	Fall	Level 3	2 days/week	Math Methods II
	Spring	Final	5 days/week	

Practice-Based Math Methods Course Context

Throughout the Math Methods I and II courses, I emphasized MP3 and the practices needed to facilitate it in the mathematics classroom. Throughout the Math Methods I and II courses, I provided opportunities for TCs to engage in McDonald, Kazemi, & Kavanagh's (2013) Cycle for

Collectively Learning to Engage in an Authentic and Ambitious Instructional Activity framework (figure 3 below) addressed earlier in chapter one. The cycle consisted of four quadrants in which TCs learned about activity, rehearsed activity, enacted activity with students, and analyzed their enactment.

Quadrant 1: Learning about the activity. For this quadrant, providing *representations of practice* (Grossman, Hammerness, et al., 2009) helped TCs to develop descriptions of what teaching behaviors look like for facilitating MP3. Grossman, et al (2009) define representations of practice as "different ways that practice is represented in professional education and what these various representations make visible to novices" (p. 258). Having an exemplar to use as a model allowed TCs to develop a way of seeing how to facilitate students' engagement in MP3 themselves. Three days during the semester, instead of class, TCs were assigned a teacher at their clinical internship school to observe. In order to observe with a purpose, TCs used a protocol based on Bostick, Matney, and Sondergeld's Standards for Mathematical Practices (SMPs) Look-for protocol (2017). The protocol (Appendix H), utilized the same indicators for MP3 as in the Bostick, Mattney, and Sondergeld (2017) protocol, including; provide and orchestrate opportunities for students to *listen* to the solution strategies of others, *discuss* alternative strategies or solution(s), and defend their ideas; *Ask* higher-order questions which encourage students to defend their ideas, consider student(s) response(s); *Provide* prompts/tasks encouraging students to think critically about the mathematics they are learning, must be related to argumentation or proving events; and Engage students *in proving events* encouraging students to develop and refine mathematical arguments (including conjectures) or proofs. TCs took field notes on teaching behaviors they noticed/observed supporting these indicators.

Quadrant 2: Preparing and rehearsing activity. For this quadrant of the learning cycle, TCs planned and enacted a lesson during methods course class time with a focus on MP3 (Appendix F). Rehearsing mathematical practice 3 within a controlled setting (university classroom), allowed for TCs to engage in *approximations of practice* (Grossman, et al, 2009). Approximations of practice "refer to opportunities for novices to engage in practices that are more or less proximal to the practices of the profession" (Grossman, et al, 2009, p. 2058).

The teaching simulation lesson plan explicitly required TCs to consider MP3 by describing teacher behavior, expected student behavior and listing questions to elicit student thinking relevant to MP3. The rest of the class assumed to be "the students" and then immediately after completed peer feedback (Appendix G) which they documented the ways they, as the student, engaged in MP3 during the lesson. Peer feedback also allowed TCs to analyze and critique the ways others were facilitating MP3 (Kazemi, Franke, & Lampert, 2009).

Quadrant 3: Enacting and analyzing the activity with students. Once TCs acquired a concept and ideas about how to facilitate MP3, TCs began "to engage in practices that are more or less proximal to the practices of the profession" (Grossman, et al, 2009) by planning for and enacting the practice with students in their clinical internship classrooms. For this quadrant in the learning cycle, TCs began an *investigation of practice* (Grossman, et al., 2009) by completing lesson plans and attending pre-conferences with supervisor which required them to; describe how they planned to implement the mathematical practices in a lesson during clinical internship and think about the impact of their actions with respect facilitating the Common Core Mathematical Practices on student learning. According to McDonald's et al., (2013) learning cycle, the last quadrant continued TCs' investigation of practice. This quadrant focused on engaging TCs in a reflection of their experiences facilitating MP3 in their clinical internship classrooms. An

important learning tool in the cycle, TCs learned from their own experiences by reflecting on their practice and providing peer feedback to others' enactment (Bailey & Taylor, 2015).

Quadrant 4: Analyzing enactment and moving forward. Quadrant four continued TCs' investigation of practice by reflecting on their experiences enacting behaviors of MP3. (Appendix L). Reflecting on experiences is an important learning tool in the cycle of learning to enact core practices (Bailey & Taylor, 2015). Reflections of teaching played a role in "unpacking teaching in ways that give students access to the pedagogical reasoning, uncertainties and dilemmas of practice that are inherent in understanding teaching as being problematic" (Loughran, 2007, p. 6). This is important for making visible the teaching behaviors and their influence on further teaching practices involved in engaging students in MP3.

Table 2 illustrates the settings, or context, where TCs engaged in the activities of practice-based methods during the cycle for this study (also considered the context for the activity system). McDonald, Kazemi, and Kavanagh (2013) stated, "setting shapes the work of teacher education so drastically, having a common framework across setting will help scholars and practitioners aggregate knowledge from diverse settings" (p. 7). Learning, rehearsing, and enacting practices took place in authentic K-5 settings and in a controlled setting at the university.

TCs observed teachers modeling practices in K-5 classrooms and again when they were enacting the practice with students in their clinical internship classrooms. The methods course classroom at the university was the site for the controlled setting where TCs rehearsed practices with their peers. During math methods class time, TCs worked with a peer to write and enact a simulation lesson focused on the mathematical practice. TCs also reflected on their experiences in observing and enacting practices in authentic settings during math methods class time.

Table 2
The Learning Cycle Across Settings

Methods Course	Setting	Quadrant	Learning cycle implementation
I	Authentic Setting K-5 School	1- Learning about practice	TCs observe K-5 teachers enact practice with their students. TCs take notes on observation protocol. TCs reflect on experience using reflection protocol. Receive instructor feedback.
I	Controlled Setting Methods course held at university	2- Rehearsing practice with peers	In-class teaching simulations. TCs work with a peer to collaboratively plan and enact lessons with a focus on MP3. Receive peer feedback.
I and II	Authentic Setting K-5 School	3- Plan and enact practice with students	TCs plan and enact MP3 with their own students while CTs provide in the moment feedback.
I and II	Controlled setting Methods course held at university	4- Analyzing enactment and moving forward	TCs Reflect on their experiences with planning and enacting with students. Begin to think about how to facilitate MP3 for all students.

Learning to teach mathematics equitably. As a math teacher educator, I consider learning to teach mathematics amalgamated to learning to teach mathematics for all students. Equitable teaching is not an additional component to learning to teach, rather, it is a "part of a system of essential elements of excellent mathematics programs" (NCTM, 2014, p. 59) and at the core of becoming a competent mathematics teacher. In learning to establish themselves as teachers, TCs should develop a focus on equity and understand their professional responsibility to ensure all students have access to mathematics (NCTM, 2014). Therefore, when I used "facilitate Mathematical Practice 3", it meant to include equitable teaching practices.

As TCs gained more experience learning about, rehearsing, and enacting core practices, they began to put the complex layers of teaching together (Zeichner, 2012). Zeichner (2012) stressed the importance of scaffolding the learning of TCs towards integrating distinct practices together.

He stated, “it seems developmentally inappropriate to start by trying to have novices master everything at once before teaching the individual components of teachers’ practice” (p. 379). As the students in the math methods gained experience in learning how to facilitate MP3, they began to think about ways to engage all students in constructing arguments and critiquing the reasoning of others. As the semester moved forward, methods course class discussions, lesson planning, observations, and lesson reflections regarding engagement in equitable teaching practices became more frequent and intentional.

Clinical Internship Context

My definitive role as a university supervisor is improving TC’s teaching practice through the facilitation of unpacking the complex layers of teaching. In addition, my aim is to support TCs in forming professional teaching identities and the agency required for longevity in the field. My supervision practice is built on my foundational previous experiences in teaching and coaching which pivot on fieldwork experiences including; fostering collaboration, building and maintaining relationships, making connections from theory to practice, modeling and promoting professional behavior, supporting reflective thinking processes, and preserving high-expectations. Sharing the belief that students learn by doing, fieldwork is important for collaboration and developing ideas of professionalism. As a University Supervisor, I spent one day a week on the clinical internship school campus holding pre and post-conference observation meetings, observing TC teaching, communicating with CTs and administration (building and maintaining relationships), and planning and teaching seminars.

CT responsibilities. CTs completed clinical educator training provided by the university and were state-certified to teach in elementary school classrooms. CTs worked closely with their TCs during each level of clinical internship. CTs and TCs co-planned and used co-teaching

techniques throughout all levels of clinical internship. CTs engaged TCs in two formal observation cycles (pre-conference, lesson observation, post-conference) for levels two and three, and three formal observation cycles during the final clinical internship. Overall levels of clinical internships, CTs supported the intern in gradually taking the lead in co-planning and co-teaching situations. CTs provided meaningful and formative in-the-moment (live) or immediate feedback.

Clinical internship school demographics. Mangrove Creek Elementary School (pseudonym) is located within 20 minutes of the TCs university in the southeast United States. The elementary school is positioned between newly developed gated communities and older neighborhoods. Approximately 30 percent of the students came from low-income families and about 6 percent of the school's population was receiving ELL services. Nearly 53 percent of the students are White, 25 percent Hispanic, 12 percent Black, 5 percent are Multi-Racial, and 5 percent are Asian. On average, classrooms have 18 students with one teacher however, there are a few co-taught classrooms (two teachers) consisting of about 30 students.

Selection of Cases

Participants for this study were chosen after grades were submitted for the methods courses and clinical internships. Seven participants were asked in-person to take part in this study fitting certain criteria including, enrolled in both my Math Methods I and II Courses, I was their Supervisor for level II, II, and final clinical internship, and taught mathematics during the final clinical internship. It was important the participant was in a clinical internship classroom allowing them to continue to teach mathematics during final clinical internship because teaching behaviors for facilitating MP3 could further develop. Additionally, I would continue to observe the ways they facilitate MP3 beyond level three clinical internship. Of the seven TCs fitting the

criteria, three TCs volunteered privately either in-person or by email indicating their willingness to participate in the study. Miles, Huberman, and Saldana (2014) suggested a small sample size for multiple case studies to allow in-depth study of people "nested in their context" (p. 31). Cresswell (1998) and Stake (2006) suggested examining fewer than four cases in order for each case to be adequately explored.

After volunteering to take part in the study, participants received a hard copy and electronic copy of the informed consent form (Appendix B) and had seven days to decide whether or not to participate. I collected hard copies of their signed consent forms and provided each with a hard copy to keep for reference.

Data Sources

In a multiple case study, it is important the data collected will "document both the unusual and ordinary" (Stake, 2006, p. 30) and take place naturally throughout the happenings of the case. Below are the methods for data collection used in this study. TCs' data were collected across both the Math Methods I and II courses and corresponding clinical internships. Interviews were conducted after submission of course grades in order to gain a better insight intoC experiences and perceptions about how the activity system components have influenced their learning to facilitate MP3. Figure 4 shows the timeline for data collection across both the Math Methods I and II courses and clinical internships.

Autobiography assignment (video and reflection). TCs come to teacher preparation programs with pre-established cultural beliefs about teaching stemming from their previous experiences in education. To explore the historical and cultural settings within TCs' individual activity systems, I looked at their Mathematics Learning Autobiography assignment (Appendix J) completed at the beginning of their Math Methods I course. The purpose of this assignment

was to have TCs reflect upon the issues and factors that influenced their mathematics learning in school and to think about how these factors may affect their teaching of mathematics to children.

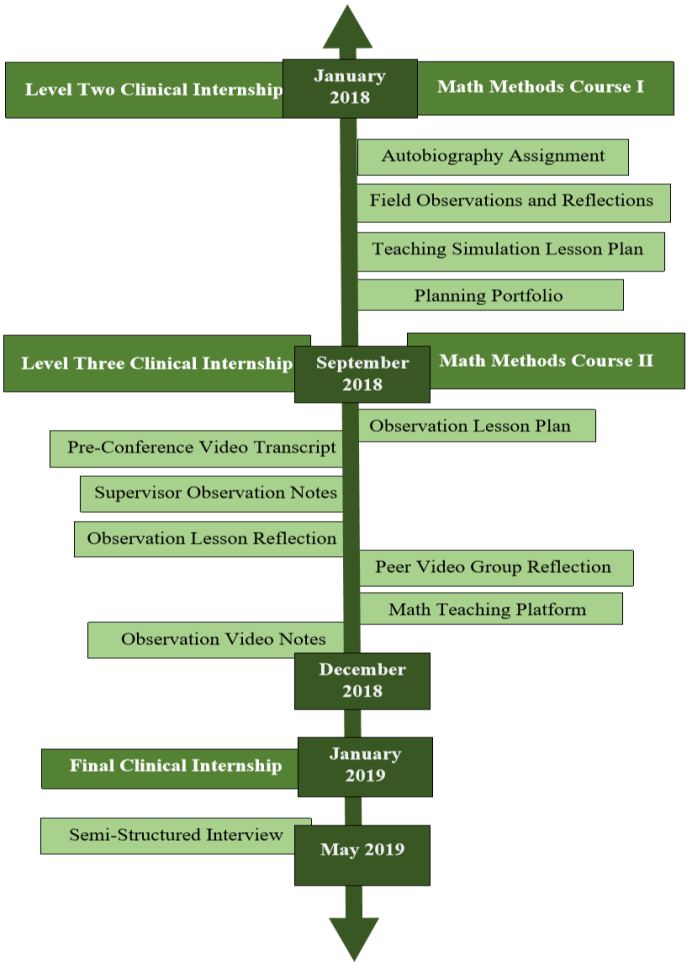

Figure 4. Data Collection Timeline for Practice-Based Methods Course and Clinical Internships

TCs' mathematics teaching platform. Also, to explore the historical and cultural settings within the activity system I looked at each TC participant's mathematics teaching platform. TCs write a mathematics teaching platform towards the end of the Math Methods II course. Their teaching platform assignment (Appendix K) was intended to capture their beliefs as a mathematics teacher of elementary school students. The platform assignment engaged TCs to

reflect on how and why they plan to teach students mathematics based on what they have learned in Math Methods I, Math Methods II, and their time in clinical internship classrooms.

Lesson plans. TC participants submitted two written lesson plans (Appendix D) for each of the two formal mathematics lessons. The formal lesson plans provided me with data as to how TCs planned and intended to facilitate MP3 and provided insight as to the knowledge they had about facilitating MP3.

Planning portfolio. TCs created a planning portfolio (Appendix M) where they described what teaching behaviors for MP3, they planned to enact in clinical internship classrooms. They then compared what they planned with what behaviors they actually enacted. Finally, they reflected on the impact of those actions on the classroom learning environment.

Peer video group reflections. For the first lesson completed during the first half of the course, TCs engaged in peer video groups where they provided and received feedback on peers' recorded videos (Appendix E). Providing feedback to peers was one-way TCs engaged in noticing and analyzing classroom instruction, specifically those core practices involved in facilitating MP3.

Field observation protocols and reflections. TCs observed with intent mathematics lessons being taught in a K-5 classroom. During the observation, the TCs took field notes using Bostic, Mattney, and Sondergeld's (2017) Look-Fors protocol (Appendix H). Following the observations, the TCs completed a reflection (Appendix I) in which they consider the key areas related to instruction and assessment.

TC's observation lesson reflections. Each TC submitted a written reflection (Appendix L) of their two lessons. Reflections provided me with data on how TCs are looking at both how they

facilitated MP3 and students' engagement of MP3. It is one way for TCs to consider what happened during the lesson and how to improve their teaching for future lessons regarding MP3.

Pre-conferences video transcripts. The observation cycle I conducted with each TC included a pre-conference to support TC in planning their observation lesson, observation, and then a post-conference to debrief the observation lesson. The second lesson was a supervisor observation, where I recorded pre-conferences of the lesson. Conferences were ways I provided one-on-one coaching for planning. Recording these conferences provided data for a deeper understanding of each TC's knowledge of the facilitation of MP3.

Notes from videoed observations. TC participants submitted a recorded video of these two lessons. As I watched the teaching videos, I took notes as to the behavior of TC's using the Bostic, Matney, and Sondergeld (2017) Look-fors Tool (Appendix H) for MP3 to denote teaching behaviors for the enactment of facilitating MP3. I watched each video three times and my notes became more detailed each time a video was watched. We (TC and Researcher) used MP3 Look-Fors (Bostic, Matney, & Sondergeld, 2017) as a basis for analyzing teacher and student behaviors consistent with facilitating/engaging in MP3. It is important to be familiar with the context of each lesson before conducting formal coding methods. The observation notes provided me insight into how TCs are enacting student engagement in MP3.

TC semi-structured interview transcripts. Interviews allowed me to dig deeper into TC perceptions of how their CTs, clinical internship classrooms, peers, and teaching beliefs played a role in their learning to facilitate MP3. Merriam (1998) states, "interviewing is necessary when we cannot observe behavior, feelings, or how people interpret the world around them" (p. 72). Similarly, Seidman (2006) asserts, "the primary way a researcher can investigate an educational organization, institution, or process is through the experiences of the individual people" (p. 10). I

employed semi-structured interviews (Appendix C) to allow for open dialogue and conversation with the ability to vary questions if needed and with expectations of receiving authentic information (Merriam, 1998; Kvale & Brickmann, 2009; Lichtman, 2013). The interviews took place after grades were submitted for math methods course and clinical internship, were audio-recorded and were about 60 minutes each in duration. Immediately following, the interviews were transcribed using speech to text online website. After receiving the text from the website, I listened to each interview at least twice while editing transcription for accuracy.

Data Storage and Protection

Pseudonyms were given to each participant known only to me and identifying markers (such as names, clinical internship schools, or university ID numbers) in the data set were removed to ensure privacy. I collected all consent forms and stored them in a locked file cabinet. I stored all digital data on a password-protected computer belonging to me and/or and physical data in a locked file cabinet in my office.

Data Analysis

The aim of this study was to explore the ways TCs learn to facilitate MP3 including planning and enacting student engagement within the contexts of a practice-based math methods course and clinical internships. I explored each case as a separate entity, attending to the configurations within the case, followed by a comparative analysis across cases attending to interpretive synthesis and common themes (Miles, Huberman, & Saldana, 2014). Looking again at the activity system for this study (figure 2), the practice based-methods course and clinical internship, I wanted to know how these components influenced TCs' facilitation of MP3 through planning and enactment within their clinical internship classrooms. Miles, Huberman, and Saldana (2014) strongly suggest synchronously collecting and analyzing data for "collecting

new, often better, data" (p. 70). It is important to note, through this exploratory case study, I intended to capture evidence of TC's planning and enaction of practices and behaviors for facilitating MP3. There is limited research and knowledge about how teachers learn to engage elementary students the Mathematical Practices. This research intended to provide evidence of a trajectory for novice teachers learning to facilitate MP3. Therefore, as I analyzed data throughout the study I focused on and described what behaviors and actions TCs exhibited (rather than what was not exhibited) and provided evidence for how each facilitated MP3.

Initial Data Analysis

Initial data analysis happened before I conducted semi-structure interviews as a way to dig deeper into the components of the practice-based methods course and clinical internship influencing TC learning. Engaging in the first cycle of coding prior to interviewing provided a holistic view and initial codes as to how TCs learned to facilitate MP3. Initial data analysis also informed the interview questions and I used data sources to stimulate recollection for TCs to drive on-going data collection (Appendix N). I addressed the patterns and themes from the initial analysis in the semi-structured interview, asked for justification and clarification, and probed TCs thinking in their facilitation of MP3. This also was used to triangulate data collected throughout the study and my understanding of how TCs learned to teach mathematics.

Individual Case

I first explored "ordinary happenings for each case" by describing how and the extent to which each TC learned to engage students in constructing reasonable mathematics arguments and critique the reasonableness of their peers' mathematics arguments. I first studied each TC's experiences separately to gain an understanding how they were situated in learning to facilitate MP3 (Stake, 2006).

First cycle coding data. I used constant comparative methods of data analysis and coded for themes representing "key concepts drawn from data" (Lichtman, 2013, p. 258). According to Miles, Huberman, and Saldana (2014), codes are categories that assign meaning to the information and "chunks" of data gathered during the study. To answer the research questions, make sense of my data, and interpret what participants have said and what I have read, I brought together, condensed, and interpreted data in a process that involved "moving back and forth between concrete bits of data and abstract concepts, between inductive and deductive reasoning, between description and interpretation" (Merriam, 2009, p. 176). First, I began with an inductive method of coding.

In order to answer and consider the influences of the components of the activity system, the practice-based methods course, I used open coding where I read through raw data multiple times (Merriam, 2009). As I read and reread through raw data, I wrote down notes and took notations of thoughts, comments, wonderings, and connections relevant to the components of the practice-based methods course (activity system) in the margins nears chunks or bits of data (Merriam, 2009). After working through the data in this way, I then constructed and assigned codes to my notes that were common and related together (Merriam, 2009). In order to capture TC's perceptions and experiences (thoughts, learning, emotions, etc.), notes combined a variety of different coding methods described by Miles, Huberman, and Saldana (2014) including descriptive, emotion, and values coding. Codes synopsized chunks of data using nouns, short phrases, gerunds ("ing"-words), and labeled participant emotions and values (Miles, Huberman. & Saldana, 2017). This was done for each set of data collected and codes were attached to each data set.

As part of the initial data analysis, I looked at the ways TCs planned and enacted the facilitation of MP3. To do this, I looked at the tools of the activity system, lesson plans, pre-conference videos, observation video notes, and TC's lesson reflections. One of the key concepts of activity theory is appropriation (Wertsch, 1981; Leont've, 1981). Appropriation denotes the learning progression through which the subject accepts and uses tools in social settings. For TCs, appropriation is a developmental process that allows them to adopt tools associated in and for teaching (Wertsch, 1981; Leont've, 1981). During the process of appropriation, learners may grasp the use of tools for various reasons. The factors contributing to appropriation can spur from the context or environment of the activity or the individualities of the learner, such as prior experiences as a student, personal goals and expectations, and beliefs and content knowledge. According to Grossman and her colleagues (1999), there are five varying degrees of appropriating tools. As I explore the tools, I will code them for evidence to the degree that TCs grasped and applied the concept of each.

Second cycle coding data. After I assigned codes to raw data, the next step, axial coding, involved grouping the open codes as they related to one another (Merriam, 2009; Lichtman, 2013). From the running list of codes gathered during the initial open coding process, I compared codes from one set of data to another, this time making a separate master list of notes, comments, and wonderings. This master list then became a preliminary classification system for the recurring patterns into which the rest of the items in the study were sorted (Merriam, 2009).

It is important at this point in data analysis that I became more deductive as I "tested" the category schemes to see if they held up as I further analyzed data. Merriam (2009) states, "As you get toward the end of your study, you are very much operating from a deductive stance in that you are looking for more evidence in support of your final set of categories" (p. 183).

Therefore, I refined, revised, collapsed, expanded these categories as the data necessitated. Each chunk of data was then sorted into these categories as evidence, preserving identifying codes (Merriam, 2009).

Third cycle coding data. The last cycle of coding specifically attended activity theory and to the third question guiding this study acknowledging any tensions that may have emerged throughout the system. I used selective coding to decide which key ideas and categories represented the data (Lichtman, 2013). These descended from at least 2 resources (e.g. the researcher, participant) and are situated in the activity theory framework presented earlier in the paper (Merriam, 2009). The activity theory framework guided the way I made meaning from the data sources analyzed. I used the influence of components within the practice-based methods course as a guide for the ways to understand the ways these TCs learned to facilitate MP3. The findings from individual cases then became a means for looking and presenting findings across the cases.

Across Cases

In a case study, participants experienced various things simultaneously and as a result, I was required to take a broad or wide lens to look for correspondence with different variables in different settings and contexts (Stake, 2006). Correspondence or covariation as Stake (2006) describes is what "things are happening together" (p. 28) and how researchers come to understand this interactivity or "the ways in which the activity of the case interacts with its context" (p. 28). Further, comparing across cases allowed me to see commonalities and differences among TC's experiences in the practice-based methods course (Stake, 2006). For cross-case analysis, the most significant findings from single cases came together and mingled to extend my understanding of the research questions (Stake, 2006). I aimed to use the components

of the activity system as a way to make sense of the commonalities and differences between TC's facilitation of MP3 in their clinical internship classrooms. Here, using the activity system model helped me to look across the three cases at once to synthesize findings and see commonalities more effectively (Miles, Huberman, & Saldana, 2014). Figure 5 illustrates the coding process for the cross-case analysis.

First, I read through each case taking notes on findings, uniqueness of each case, and relevance of themes. Then, I used constant comparative methods to look across the themes and patterns of each activity systems in which TCs facilitated MP3. The comparisons across each of TC's activity systems were guided by the following questions: What commonalities and differences are evident in the themes, relationships, and patterns across the three cases? How do these commonalities or differences explain the ways TC's facilitate MP3? What influences of the components of the activity system may explain these commonalities and differences?

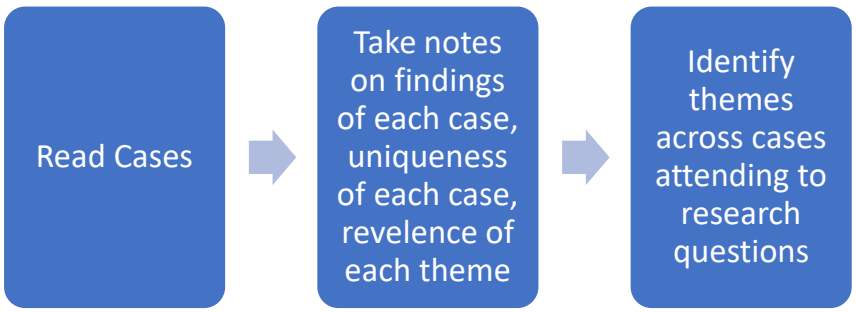

Figure 5. Cross Case Data Analysis

Quality

Stake (2006) asserts "qualitative studies are intentionally value laden. Their credibility and utility will be influenced more by even-handed treatment than by statements or implications that the report is free of advocacy" (p. 85). The choices and decisions I have made throughout this study are clear and supported by researchers and methodologists in the field, such as Stake,

Merriam, Miles, Huberman, & Saldana. The following criteria and standards for conducting quality research were considered in this study, which were driven by Lincoln and Guba's (1985) quality criteria and defined by those mentioned above.

Dependability/credibility. Extreme care was used in carrying out this study including attending my role as a researcher (Miles, Huberman, & Saldana, 2014). Additionally, I looked towards theory and theoretical frameworks as building blocks for this study. Data were collected across a variety of settings, contexts, times and participants (Miles, Huberman, & Saldana, 2014). Establishing credibility, or validity, addressed issues of "truth value" and considered "thick" descriptions and triangulation (Geertz, 1973; Tracey, 2010; Miles, Huberman, & Saldana, 2014). In aiming to achieve and maintain a high level of credibility, I was transparent about my positionality as a math methods instructor and university supervisor. These roles do influence the work that I do (Janesick, 2011; Stake, 2010), therefore I was explicit about my role as a researcher with my participants. I ensure that participants were chosen after grades were assigned to make clear to potential participants that their participation (or lack of participation) in no way affected their grades in the course. I developed a trusting relationship with participants in this study and provided a mutual space for interviews where they felt comfortable sharing their experiences and perceptions with me.

Triangulation. While I subscribed to the constructivist paradigm and believe there may be multiple perspectives in the representation of the data, I wanted to provide readers "with good raw data for their own generalizing" (Stake, 1995, p. 102). Therefore, I used "thick" descriptions (Geertz, 1975) as a way to "present a substantial body of uncontestable description" (Stake, 1995, p. 110). Also, I ensured that I generated meaningful interpretations by using multiple methods of data sources, looking at it from multiple angles, and gathering data repeatedly over a

period of time, in this case, the course of the methods course (Stake, 1995; Merriam, 1998). The purpose of triangulation is to "see if the phenomenon or case remains the same at other times, in other spaces, or as persons interact differently" (Stake, 1995, p. 112). Therefore, I used multiple data sources to answer my research questions. The following table (Table 3) illustrates how the data sources were used for confirmation of information.

Transferability. Although case studies "look for detail of interaction with its contexts" (Stake, 1995, p. xi), and are often not considered being generalizable, Stake (1995) asserts that if enough details are provided in describing the study, then it can hold naturalistic generalizations. Careful attention to describing frameworks and theories, the context of study, inclusion and exclusion criteria for participants, and processes for data collection and data analyzation can provide transferability of findings.

Limitations. Considering I was the math methods course instructor, university supervisor, and researcher for this study, I fully presented my beliefs (biases) as a math methods course instructor, university supervisor and research above in the Role of Researcher Section. Additionally, I acknowledged these roles may have caused the participants to hesitate to share with me their perceptions and relied on many data sources to confirm findings that answer my research questions. Further, I acknowledge that my presence for observation lessons may have forced a certain behavior not normally executed during lessons as TCs knew I was focusing my research on MP3.

Table 3
Triangulation of Data Sources

	Components of Activity System							
Data Collection	Plan	Enact	Cultural Context	Tensions	Tools	Division of Labor	Rules	Community
Autobiography Assignment (Video and Reflection)			X					
Teaching Platform			X					
Lesson Plans (2)	X				X			
Planning Portfolio								
Peer Video Group Reflection		X			X			X
Field Observation Protocols and Reflections (3)								X
Observation Lesson Reflections	X	X	X	X	X	X	X	X
Pre-Conference Video Transcript	X			X	X	X	X	X
Lesson Observation Video Notes		X	X		X	X	X	X
Interview Transcript	X	X	X	X	X	X	X	X

Chapter 4: Findings

In this chapter I present each of the three cases individually, attending to the research questions guiding the study followed by a chapter presenting a cross-case analysis. For this chapter, each case begins with the TC's background information, specific clinical internship contexts, and coding details including a codes table. As each TC is the subject of their activity systems, which is cultural in nature, it is important I include information which may influence the outcome, or in this study, their facilitation of MP3. Therefore, each case begins with background information related to TC's prior experiences learning math and specific contexts associated with their clinical internships. Following TC's background information, a codes table and overview of how they emerged from the data analysis are presented for each case. Thereafter, each case is presented according to research questions, first addressing how TCs plan to engage students in MP3, then how they enact teaching behaviors for facilitating MP3 within their clinical internship classrooms, and finally, supports and tensions related to facilitating MP3. The planning and enacting sections open with a vignette to illustrate a holistic overview of the typical conversations and observations with each TC. Due to my role as supervisor and math methods instructor to the TC's throughout their time in the teacher preparation program, I had broad access to their coaching and observation experiences and the vignettes allowed me to draw on these experiences to provide a thick description and intentional portrayal of TC's facilitation of MP3. Following the vignettes, a description of how each TC's

facilitated MP3 is supported using thick description and evidence from the data set. The last section describes the tensions and challenges as perceived by the TC's as they facilitated MP3. Finally, each case is concluded with a summary.

Case 1: Julianna, The Questioner

Background and teaching beliefs. According to Julianna's Autobiography Video and Reflection, she revealed she did well in elementary math classes and even was a participant in the Math Bowl Team in 5th grade. She thought math would be easy however, after elementary school, math began to get harder and she struggled while doing the bare minimum in classes. This is when she started to dislike math. Julianna expressed that she was not actively engaged in math class as a middle/high school student. She admits to being anxious about teaching math because she feels she has limited content knowledge as a result of her experiences learning math. She also knows she will be teaching math differently than how she learned it with different methods and strategies.

According to Julianna's Mathematics Teaching Platform, she aligns with a constructivist pedagogy where she aims to use collaborative teaching methods for students to use "multiple interpretations and expressions of learning (p. 2). Further, Julianna intends to create a classroom to encourage a productive mindset and develop independent learners "where my students' mathematical problem-solving skills can flourish" (Teaching Platform, p. 3). She hopes to achieve this through activities requiring students to use "adaptive reasoning skills and conceptual understanding of mathematics" (p. 3). In creating a classroom environment that inspires students to reason logically and use visual representations to find solutions to math problems, Julianna hopes to ensure "my students are presented the best opportunity to learn" (Teaching Platform, p. 4).

Clinical internship contexts. Julianna's level three clinical internship was in a third-grade classroom where her CT used homogeneous ability-leveled small groups for teaching math with limited whole-class instruction. During final clinical internship, Julianna was assigned to a second-grade classroom where her CT used collaborative methods in teaching whole class, small group, and partner work for math. Julianna attended and participated in weekly grade level math planning meetings. It is important to note that Julianna had a close relationship with her CTs in all of her clinical internship assignments. They communicated through email and text for personal and professional reasons both inside and outside of school time.

Codes table. The following codes table (table 4) was created after analyzing 86 pages of data from Julianna's autobiography assignment, planning portfolio, math teaching platform, field observation notes and reflections (3), peer video groups notes and reflection, lesson plans (2), supervisor observations notes, supervisor observation video notes, pre-conference video transcript, observation reflection, and interview transcript. I began data analysis with open coding where I read through the raw data multiple times. As I read and reread through raw data, I wrote down notes and took notations of thoughts, comments, wonderings, and connections relevant to the facilitation of MP3 in the margins nears chunks or bits of data. After working through the data in this way, I constructed and assigned color codes to my notes that were common and related together. There were eight assigned codes at this time including, student support (SS), deeper understanding of math (DU), share/listen/compare (SH-L-C), in-the-moment (ITM), monitor (M), question (Q), facilitate discussion (FD), and activity theory (AT) (Appendix O).

After I assigned codes to raw data, the next step, axial coding, involved grouping the open codes as they related to one another. From the running list of codes gathered (above) during the

initial open coding process, I compared codes from one set of data to another, this time noting comments, thoughts, and wonderings as to the emerging patterns. I used jottings (Miles, Huberman, & Saldana, 2014), drawings, and doodles to record emergent thoughts or ideas coming to mind as I reflected on the data. It is important at this point in data analysis that I became more deductive as I "tested" the category schemes to see if they held up as I further analyzed data. The process of testing codes included refining, revising, collapsing, expanding, and naming them as the data necessitated.

Question (Q) was the most frequently occurring code throughout Julianna's data and is why I have labeled her as the Questioner. When collapsing, refining, and revising initial codes, I noticed that questions (Q), facilitate discussion (FD), supporting students (SS), In-the-moment (ITM) decision-making, and monitoring (M) codes could be collapsed into a broader category of facilitating discussion. Looking closer at these codes within facilitating discussion, I perceived differences in purpose, timing, and types of questions asked during discussions. As the codes became more distinct, I saw patterns emerge for ways Julianna structured whole class and small group discussions around MP3 focusing on supporting students (SS) and questioning (Q).

Codes labeled as sharing (SH), listening (L), and comparing (C) were combined into the larger named category of providing an opportunity for students to share, listen to, and consider the solutions of others (SHLC). Finally, each chunk of data was then sorted into these categories as evidence preserving identifying codes and I created the final codes table below (Table 4).

The next sections address each research question separately and describe how Julianna facilitated MP3. The first two sections describe Julianna's planning and enacting teaching behaviors and practices for engaging students in MP3 and each begins with a vignette to illustrate an overall picture that draws on her experiences and data analysis. I used a vignette to

illustrate a holistic overview of the typical conversations and observations with Julianna. Due to my role as supervisor and math methods instructor throughout Julianna's time in the teacher preparation program, I had broad access to her coaching and observation experiences and the vignettes allowed me to draw on these experiences to provide thick description and intentional portrayal of her facilitation of MP3.

Table 4
Julianna's Data Analysis Codes

Code	Descriptors
Provide Opportunity for Students to share, listen to, or consider the solutions of others (SHLC)	See and encourage use of multiple strategies Position students as authors of ideas who explain and justify their work (whole class and partner) Intentional Selection and sequencing student approaches for whole class analysis Decide if others' solutions make sense Correct vs Incorrect Solutions Correct Solutions Teacher Modeled Solutions
Facilitating Whole Class and Small Group Discussion (Q)	Pattern Structure for Whole Class Discussion Facilitating Small Group/Partner Discussion Translator for Supporting Language Questioning Clarifying and Probing Questions to reveal a deeper understanding of student reasoning and actions Procedural Questions for Supporting Struggling Learners Reflective Questions Types and patterns of questions
Activity Theory (AT)	Supports CT Observation, Coaching/Feedback, and Clinical internship Classroom Clinical internship School District Curriculum Practice-Based Methods Coursework Observations Practice Reflection Peer Video Groups Supervisor Conferencing and Coaching Tensions Student Behavior Language and Student Ability Levels

Research question 1: How does the TC plan to facilitate MP3? *As the University Supervisor, I am enjoying the tranquility of the book stacks when the door suddenly opens to Julianna's smiling face. Julianna is in the first semester of her senior year and a level 3 intern which allows her to be in a clinical internship classroom two days a week.*

"So, tell me about your lesson," I ask. Julianna briefly informs me which lesson from the curriculum she will be addressing. We have the typical discussion about the standards and the objectives she has written. This leads us into a conversation about her choices for formative assessment and how she will measure student learning.

Julianna indicates, "we will have eight minutes for the application problem, with the first four being for them to complete the problem independently. During this time, I will walk around and see how they are solving it. I will assist any student I see struggling during this time. I will also notice the strategies being used."

"So, you will mentally take note of what strategies students are using during this time?" I ask. I am happy to hear that she is intentionally looking for different strategies. She responds,

"Yes, I will take note of different strategies that I see and we will talk about multiple ways of solving the problem, ummm, then I will model how to solve it, and then that is the other four minutes of the application problem."

"Will any of the students be sharing their strategies?" I ask wondering why she doesn't have her students model their strategies for solving the problem instead of discussing them.

"I will ask the class if anyone solved the problem a different way and I will choose students to explain their strategies that I noticed when I walked around. I will ask, can you explain your strategy to us? And that is when we typically do a thumbs up if you agree or thumbs down if you don't agree." Julianna adds that she uses thumbs up/thumbs down strategy when she

intentionally has a student share an incorrect strategy. She assures me that she only does this if she notices that several students in the class are showing a common misconception and she wants to address it with the whole class. "I have done that with the thumbs up/thumbs down and then I'll see students give a thumbs down and I ask, why do you disagree, what do you see and then they will bring something to the classes attention." She brings up an important idea, I think to myself, critiquing the work of others allows for a deeper understanding of math and involves not only how a solution to a problem (in part or in whole) is correct, but rather how a solution is incorrect (and knowing how to make it work).

"Yes! Talking about how a problem is wrong is just as important as talking about why a problem is right so it is important to take the opportunity for you to bring attention to common misconceptions and highlight this in your math class!"

The first research question was designed to describe the ways TCs plan and highlight teaching practices/ behaviors intended for the facilitation of MP3. The primary sources of data to answer this question spanned Math Methods Courses I and II assignments including lesson plans and planning portfolios. Additionally, level three observation pre-conferences with supervisor (myself), observation lesson plans and interviews were included as sources. The data traversed the TC's time in the program (program context described in chapter 3) and thus provided a trajectory for their development for thinking about MP3 within both clinical internships and Math Methods coursework from level 2 interns to final clinical internship.

Math methods course I and level two clinical internship. Planning during this time for Julianna included providing questions that could be generalizable to any math class and not specific to any one class or student. In Math Methods Course I, she wrote questions such as "ask students to explain solutions" (Planning Portfolio, p. 1 line 12) and teaching behavior "while

walking around and observing asking students why they think ____ is the answer and how they got that answer" (Planning Portfolio, p. 1, lines 13-14). Julianna did not provide details as to how she would carry out these actions, for example, she did not provide what questions did she plan on asking to have students explain solutions and what was she looking for when walking around monitoring students working.

Math methods course II and level three clinical internship. As noted in chapter three, Math Methods Course II is taken during the first semester of TC's senior year. Concurrently, TCs completed level three clinical internship in which they spend two days a week in their clinical internship classrooms. Julianna's lesson plans and observation pre-conferences provided evidence that she was beginning to plan questions, not only to think deeper about the math content but also think about and respond to the work of their peers. For example, Julianna wrote questions such as, "how did you not use addition?" (Peer Video Lesson Plan, p. 2, line 3) and indicated she would engage her students in a "thumbs up, thumbs down" strategy (Peer Video Lesson Plan, p. 2, line 6-7) for agreeing or disagreeing with others' work and solutions. Additionally, she would use "think pair share strategy to have students talk through solving solutions with one another" (Supervisor Observation #1 Lesson Plan, p. 4, lines 6-7).

She also intentionally planned opportunities for students to be exposed to different strategies by "taking note of different strategies that I saw and we will talk about the multiple ways of solving the problem" (Pre-Conference Script, p. 3, lines 19-20) and opportunities for students to critique solutions (both correct and incorrect solutions) by looking for when students "put a thumbs down, and I'm like why do you disagree, what do you see and then they [student] will bring something to our attention and we will talk about how it [the solution] was wrong" (Pre-Conference Script, p. 4, lines 11-13).

Final clinical internship. During final internship (second semester senior year) when Julianna spent five days a week in her clinical internship classroom, planning for the facilitation of MP3 became more focused on responding to students in-the-moment. If I am espousing that facilitation of MP3 involves core practices as earlier defined by Macdonald, Kazemi, and Kavanaugh (2013) and attends to in-the-moment decision making, then it makes sense Julianna's planning would reflect when and how to engage students in MP3.

During final internship, Julianna expressed facilitating MP3 is essential for engaging her students in deeper learning and a component of effective math teaching. However, it is not something she explicitly planned for because she relies on responding to students and requires in-the-moment thinking. When asked if she planned for the facilitation of MP3 Julianna responded, "no, honestly I do not [plan for MP3]. It's in my head, like, I know like I'm going to have to [facilitate MP3], I'm going to ask them [students] why, like what triggered you to do this" (Interview, p. 6, lines 1-2). Julianna also indicated how she thinks about responding to students, "I look at student responses, I look at how they're doing with the lesson, and then I come up with ways to make sure they are understanding and I kind of break it down throughout the lesson, but I don't intentionally plan for MP3 during my lesson planning" (Interview, p. 6, lines 6-9). She also explained her in-the-moment thinking when responding to her students during the lesson when reflecting on a lesson by stating,

> During the lesson, I expected many of my students to use different strategies while solving different problems. However, as I walked around and was quickly doing a formative assessment, I noticed a lot of students were using the same methods. This made it a challenge when having students come to the board to explain the strategy they chose to use. However, when this happened, I stepped in and displayed a different strategy for

> the students to see. I also used this as a time to explain that it's okay if our strategies are the same or different than our classmates, we need to use that works best for us! (Supervisor Observation 3 Reflection, p. 1, lines 2-10)

Additionally, Julianna described engaging students in MP3 is largely dependent on student behavior in solving math problems and knowing when to engage students in MP3 during lessons,

> Well, I think it depends on your kids, but like for my kids especially like word problems, I want to know why you chose that - what in that problem told you to add? Like, where did you get that thought process and how did you know this was a part, part, whole or you what know what, how did you know that? So that they get the practice of understanding what MP3 like you know if I am adding, this is what I am looking for or if I'm subtracting this is what I am looking for (Interview, p. 6, lines 17-25)

This quote from Julianna's interview captures her in-the-moment ability to use questions to elicit student thinking and foster math teaching through problem-solving. During final clinical internship, Julianna's planning for MP3 became more focused on being responsive to student actions in her class and knowing what to do if students are struggling, not using a strategy she wanted them to use, or using questions to push students thinking.

In summary, Julianna's planning to facilitate MP3 began with providing actions that could be generalizable to any math class and not specific to any one class or student and progressed to relying on in-the-moment decision making based on student behavior. Julianna's planning seems to suggest that as she began to learn overarching ideas about what it means to engage students in MP3, forming arguments about one's own work and consider the reasoning of others' work, she began to note those ideas into her lesson plans. For example, Julianna indicated in Math I course she intended to "ask students to explain solutions" (Planning Portfolio, p. 1 line 12). As Julianna

learned more about the practices involved for facilitating MP3, she provided further detailed action for carrying out these practices and specifically how they apply to her students. As evidenced in her lesson plans during Math Methods II, she indicated types of questions to ask, strategies for having students share ideas, talk with one another and compare answers, and approaches for students to see many ways for solving problems. During final clinical internship, Julianna demonstrated how her actions depended on students' behavior. She planned questions to support struggling students and how she would intentionally model specific solution strategies in the event students didn't demonstrate it themselves.

Research question 2: How does the TC enact teaching behaviors for the facilitation of MP3? *Julianna has already started her lesson when I sneak in and stealthily find an empty chair in the back of the classroom. My presence goes unnoticed as the class is used to seeing me come in to observe Julianna over the last 4 months. I arrive just in time to hear her reading a math problem to the class.*

"Marc and Melissa both measure the same marker with an inch tile but came up with different lengths. Circle the student work that is correct and explain why you choose that work". The students are then instructed to work together in their table groups to figure out which student is correct. She reminds them to make sure they explain why they think that student is correct to their tablemates.

As the students begin to work a buzz fills the classroom. Julianna scans the room and slowly snakes her way around the groups keeping a slight distance. She stalls briefly around each group and I can tell she is eavesdropping on their conversations. She pauses at one group, crouches down to their level, and listens for a short moment before she asks, "why do you think Marc is correct?" She listens intently as one student in the group explains why they think Marc is

correct. Julianna purses her lips and says, "hmmmmm, let's think about it" as if she wanted them to think with her. She pauses and then says, "Remember when we talked about measurement? With centimeters what is the space between the lines is called?" One student responds with interval. "So now let's look at this problem again with that viewpoint." She points down to their paper as she continues, "If I go from here to here and it is one inch, then how many inches do I have?" Julianna pauses and lets the group process what she has asked. The group members begin talking with one another and Julianna walks away slowly with a smile on her face. She begins a similar conversation with another group.

After some time and working with other groups, she brings the class back together, "Okay, we are coming back together in 5- 4- 3 -2 -1, show me a thumbs up or thumbs down, who thinks Melissa is correct?" She looks around and notices some students have their thumbs up. "Okay, show me thumbs up or thumbs down, who thinks Mark is correct?" Again, she looks around to see who has thumbs up or down. She acts surprised to discover that there seems to be an equal number of students support Melissa as Mark. She calls on a student to explain to the class why she thinks Mark is correct. The student explains that Mark is correct because they counted 7 lines (similar to a way you would count a number line). Julianna labels the lines 1-7 in the picture showing Mark's work. She chooses another student to explain why they think Melissa is correct. The student explains it's not the lines but the spaces between those lines that need to be counted.

"So, its 6 inches" the student answers. Julianna numbers the spaces 1-7 in Melissa's picture. Julianna steps back and asks the class again who they think is correct. This time, most students choose Melissa and she picks a student who has changed their mind from supporting Mark to support Melissa's answer.

"Why did you change your mind?", she asks the student.

"Melissa is correct", the student confidently states that when measuring with a ruler, the first line is actually zero and not one and therefore, the last mark would be 6 and not 7.

Julianna then corrects Mark's work on the board and explains that the next part of the lesson is going to be exploring this concept while working with rulers.

This second research question was designed to describe the ways TCs engage in teaching practices and behaviors for facilitating MP3. The primary sources of data spanned the Math Methods II Course and level three and final clinical internships including supervisor observations, TC observation reflections, peer video group notes and reflections, and interviews. During this time, Julianna progressed from having a predetermined set of behaviors towards actions that were in-the-moment responses dependent on student behavior. By final clinical internship, Julianna expressed,

> as I walked around I noticed a lot of students were using the same methods. This made it a challenge when having students come to the board to explain the strategy they chose to use. However, when this happened, I stepped in and displayed a different strategy for students to see. I also used this time to explain that it's okay if our strategies are the same or different than our classmates, we need to use what works best for us! (Supervisor Final Internship Observation 3, p. 1, lines 4-10).

This quote provides evidence of Julianna's in-the-moment decision making for providing a model for students, providing them opportunities for them to see and encourage the use of multiple strategies.

Opportunities for students to share, listen to or consider the solutions of others. Julianna provided her students opportunities to work collaboratively to share and listen to others by engaging her students in both partner and group work. Group work, such as table talks and partner work, such as turn and talks allowed students to work together on a problem to discuss different ideas and agree on a common solution. Turn and Talks provided students opportunities for sharing and comparing ideas with a partner after independently trying to figure problem solutions out. Julianna explained, "I had the students work in pairs where they each used a different strategy to solve the same problem" and "after solving the problem, the students turned and talked with their shoulder partners to discuss which strategy they used and why. This allowed for the students to discuss mathematics and different strategies" (Supervisor Final Internship Observation 1 Reflection, p. 2, lines 27-28 and 8-21). Julianna also used a Thumbs up, Thumbs down (why do you agree, why do you disagree) as a way to have students compare ideas and critique the work of others. She explained the importance listening plays in facilitating MP3 particularly as it applies to students attending to the math ideas of one another. She described,

> Sometimes I also ask, tell me what your partner said. So that holds them accountable in response to, do I understand what my partner is saying? Am I listening to them? That seems to work too because they have to listen in order to be able to share out what did your partner does for this strategy. How did they solve it? And that's' listening to their ideas. And then after that, I'll say, now how did you solve it? Did you do it the same as your partner or did you do it differently? So, listening to one another, and in that whole group, when everybody is explaining and walking us through how they did it on the board, everybody else is listening to the process. They are watching how it's done. And they thumbs up if they agree, thumbs down if they disagree. So, they're listening to the

process in order to come up to the conclusion if they agree or disagree (Interview, p. 9, lines 18-34).

This quote provided evidence for the ways Julianna purposefully used listening techniques for students to share, compare, and make sense of their peers' mathematical ideas.

Multiple ways to solve problems. Julianna had supported the idea that having students see multiple ways to solve problems is important for student learning and "taught students a method that they carried over into their individual work" (Peer Video Group Reflection, p. 2, line 19). She reflected,

> I learned how important it is to realize that students see things differently and different strategies work for different students. I think this is extremely important to realize so that we can effectively teach our students and ensure that they are learning in a way that works best for them (Supervisor Final Internship Observation 1 Reflection, p. 3, lines 11-14).

She called on students both to model strategies in whole class situations. She intentionally chose students to address misconceptions and model a more effective way of solving problems. She also provided opportunities for students to see strategies with teacher modeling, "Then, as a class, I modeled to my students another way to solve a problem and explained why I chose that way. This was whole class, therefore, each of my students were a part of this opportunity" (Supervisor Final Internship Observation 3 Reflection, p. 2, lines 34-37).

Julianna also described how providing students to see and use a variety of strategies supports struggling students by going "over it on the board so they can see it being done and then we walk through each step and that helps a lot of lower students as well because they're seeing it step-by-step, but then you are still challenging other students who need it" (Interview, p. 9, lines 1-5).

Julianna expressed providing incorrect strategies to her students allowed them to use higher-level thinking to critique and reason about why it is incorrect rather than correct,

> everyone learns the material in a different way; therefore, I value each of my students' responses. If I know one of the strategies won't necessarily provide an accurate solution for the problem, I will still follow through using that strategy, so my students can see that it is not an effective strategy and why (Supervisor Observation 2 Reflection, p. 2).

For Julianna, intentionally exposing her students to share, listen to, and consider multiple ways to solve problems is important for addressing student misconceptions and a deeper understanding of math concepts.

Facilitating whole class and small group discussion. Engaging students in MP3, Julianna relied on facilitating discussion grounded in questioning techniques. She used questions to probe student thinking, support struggling students, and have students reflect on their work.

***Structure for facilitating whole-class discussion for MP3*.** Julianna described the facilitation of MP3 in whole-class math instruction as a time to,

> discuss different strategies the students use to solve a problem and to discuss which strategies were effective and which were not. If some students used addition instead of subtraction, or whatever the case may be, we discuss our thought process and why. Then I will address any misconceptions I hear throughout the discussion and model the correct way the problem should be solved (Peer Video Group Lesson Plan, p. 2, lines 8-13).

In a whole-class instruction, Julianna's structure for facilitating discussion addressing MP3 suggested a pattern I have labeled Question-Observation-Share-Prompt (QOSP) and illustrated in Table 5.

Question. Q signifies posing a question or task to whole class such as, "who can show me by show of hands who thinks Melissa is correct? (Video Notes, p. 1). The question launching whole class discussion can also be contextual math tasks such as the problem with Marc and Melissa described in the vignette.

Observation. O represents observation because Julianna then waits and observes action responses from students, for instance, by either show of hands or holding up personal whiteboards. In this time, Julianna also monitors students' work deliberately looking for solutions to be shared. If she notices misconceptions, she will want to address them as a whole-class. Additionally, if she does not observe a certain solution, she intentionally wants to highlight then she will want to model that herself.

Share. Next, *S* denotes share because then she invites student(s) to share their ideas and respond to the question either with peers or with the whole class. Sharing with partners included "turn and talk with their partners to discuss which strategy they used and why and allowed for the students to discuss mathematics and different strategies" (Supervisor Final Internship Observation 3, p. 2, lines 19-21).

When sharing for whole-class critique, sometimes students stay at their seats and sometimes she has them (or herself) "come to the board to show their different strategies and explain it to their classmates" for analysis and discussion (Supervisor Observation 3, p. 1, lines 25-26). Julianna often encouraged her students to use visual representations/manipulatives with their explanations because it "best supported my students' development of reading and solving mathematical word problems (Peer Video Group Reflection, p. 1, lines7-8). Julianna used Thumbs Up, Thumbs Down strategy for asking the class whether they agree or disagree with the students' solutions. She may call on students to ask why they agree or disagree, "I see you are

shaking your head and you have a thumbs down, why do you disagree?" (Supervisor Observation Notes) She may call on more than one student to share their ideas.

Table 5
Julianna's Pattern Structure for Discussion in Facilitating MP3 (QOSP)

Segment	Description	Example Evidence
Question (Q)	questions or contextual math tasks	"who can show me with a show of hands who thinks Melissa is right?" Marc and Melissa both measure the same marker with an inch tile but came up with different lengths. Circle the student's work that is correct and explain why you choose that work"
Observation (O)	Provides students wait time to answer questions and observes action responses	Show of hands or use of personal whiteboards to show answers Monitors student work to notice misconceptions or strategies she wants to highlight during sharing Decides if she needs to model any strategies no used by any students
Share with whole class (S)	Students provided opportunity to share math ideas in partners or for whole-class analysis	"Turn and talk with their partners to discuss which strategy they used and why" "come to the board to show their different strategies and explain it to their classmates" Use thumbs up/thumbs down "I see you are shaking your head and you have a thumbs down, why do you disagree?"
Probe/Prompt (P)	After presenting ideas students are probed or prompted to support in explanations of solutions	"What do you think Mark is correct?" Breaking it down and just asking students to explain what they did "How did you solve this?" "Why did you choose to solve it this way?"

Prompt. Each student presenting their ideas is subject to probing and/or prompting questions, therefore the last piece of the structure is labeled *P*. This describes when Julianna prompted or probed students by asking why or how questions to support them in explaining their solutions. She may ask more than one question to continue probing students such as "why do you think Marc is correct?" (Video Notes, p. 1). For Julianna, probing and prompting students becomes a way for her to teach her students how to explain and justify their solutions by "breaking it down and just simply saying, explain to me what they did. That way they understand what I'm asking and they are able to justify their answer to me by explaining their process" (Interview, p. 8, lines 23-27).

Structure for facilitating small group/partner discussion for MP3. The prompting and probing questions Julianna asked during the last part of the pattern described above rely on in-the-moment thinking to decide whether she needs to ask probing questions to learn more about student thinking or prompting questions to support students in explaining or justifying their answers. They require Julianna to listen to students, make sense of their work and language, and respond/provide feedback for a deeper understanding of the math content in the lesson. Julianna explained she can scaffold students' responses to justify their answers by "asking probing questions like how did you solve this or why did you choose to solve it this way?" (Interview, p. 4 lines 27-28).

For Julianna, asking these types of questions aligned with teaching with problem-solving, "I want to know why you chose that, what in the problem told you add? Where did you get that thought process and how did you know that you were adding?" (Interview p. 6, lines 18-21) and she explained problem-solving is much more than computation, rather, it is knowing when and what to compute without having to be told to do so,

> I think it's important for kids to understand what they are doing and not just to be told, okay we are adding here. I want them to know when they are supposed to add, they aren't told they need to add in word problems. Sometimes students get stuck because they don't know what to do. So, it's teaching them the thought process of, I see this word or phrase and that triggers addition or subtraction. It's different because in worksheets they see the addition sign in a number sentence, but when it's put into a word problem, they don't know how to justify their answers (Interview, p. 6-7, lines 32-43 and 1-3).

This quote also aligned with Julianna's beliefs about developing independent learners as stated in her Math Teaching Platform, "I plan on creating a classroom where my students' mathematical problem-solving skills can flourish" (Mathematics Teaching Platform, p.3).

Procedural questions to support struggling learners. In working with students in small groups or one-on-one, Julianna used questioning to help struggling learners to solve and explain math problems. She begins to help students by asking procedural questions (questions that help students with the process of math problems). She explained, "if I notice students are struggling, I'm, going to ask them more questions of breaking down the problem and how do we start this process" (Interview, p. 5 lines 26-28), while "having students explain their answers is a challenge sometimes, it's hard to get them thinking conceptually rather than procedurally so I say, I know this works, but how does this work?" (Interview, p.3, lines 21-26). She used probing questions to support students to recall information or in facilitating procedural math steps such as, "What is the problem asking? What are we trying to figure out? "What did we say? How long is this paper clip? Raise your hand if you think this paperclip is 1 inch? What is five tens? What would this point be? What would you do next?" (Video Notes, p. 2).

Julianna encouraged students, but specifically struggling students in engaging in MP3 by asking probing questions to help them to think about the step-by-step process in solving math problems. This quote provided evidence to support that scaffolding is needed to help students verbalize their ideas about mathematics. Julianna supports her third graders' conversations held around math ideas and using math language by asking questions around processes for solving problems.

***Probing questions to justify/explain solutions*.** Julianna also suggests questioning allows her to support students in explaining their ideas, probing them to think about their solutions and how they came up with them.

> But there are also kids that say, I know because I know it's four plus four, but how do you know? So, sometimes they are the ones that I can kind of probe out a little bit more or I can ask the higher-level questions to get students to see things because I know that they understand it and they follow along. (Interview p. 12-14, lines 16-44,1-3)

Julianna asked questions to students who can verbalize their explanations and how they came to their solutions by prompting them to think about why their solution is correct and support them to think deeper about the mathematics in the problem.

Reflective questions. Reflective questions urge students to think about their work in a reflective way and tend to be higher-order questions beginning with why or how did you know. Julianna asked such questions such as "How can they both be right?, I see you shaking your head and you have a thumbs down, why do you disagree?, Can you explain why you chose that answer?, Why did you add/subtract?, Does this make sense?" (Peer Video Group Notes, p. 1-2).

Again, Julianna demonstrated the ways in which she attended to students to think deeper about the math and reason about their solutions. These types of questions go beyond explanation

for how students got their answers and were aimed to have students begin thinking about justifications for their work.

***Julianna as a translator*.** One interesting code I noticed emerge from the data indicated how Julianna attended to the language barriers in her classroom. Julianna rephrased or re-voiced students' responses in a way students could understand, she explained, "when students explain their answers using math language, not all my students understand. But that's when I modify it. I had to modify what they say for everybody to understand" (Interview, p. 13, lines 40-44 and p. 14, lines 1-3). She also modeled, reinforced, and encouraged all students to use math language. She encouraged all students to use math language. She expected it from those she knew were capable of using math language, "if I know a student can, then I expect it from them" (Interview, p. 13, line 13).

Additionally, for students that may be struggling with verbalizing their ideas or using math language, she understood their difficultly of using math language in their discussions. She stated, "I know it's hard for them and I just need them to tell me in their own way what they did and I will reinforce some math language in that conversation" (Interview, p. 13, lines 23-30). She provided an example of a time when she reinforced math language in her clinical internship classroom when talking about fractions and using "denominator" instead of "bottom number",

> Sometimes they say "the bottom number" and I'll respond, what is that called? And they'll say denominator. But to reinforce the word, I will say the bottom number and point to that number on the board and then I will follow up with saying denominator (Interview, p. 14, lines 30-43).

She explained she used the language provided by students (bottom number) while simultaneously modeling math language (denominator).

Research Question 3: In what ways do TCs perceive supports and tensions within their activity system?

Learning that happens in an activity system is dependent on several components and supports achieving certain goals in collaboration with others. In looking at the factors influencing facilitating MP3, the components of practiced based learning came into focus (figure 6). Viewing Julianna's teaching through an activity system lens highlights components that support the planning and facilitation of MP3 including intentional observations, CT, internship classroom environment and classroom community, and opportunity for working with children over time. Tensions that exist within the system and that were barriers to facilitating MP3 also emerged. Student behavior and ability levels, and language proved to be challenges for Julianna's facilitation of MP3.

Supports. Julianna's CT and her internship classroom played an important role in facilitating MP3 as observing, practicing, feedback (in-the-moment), and conversations with CT based on the practice that happened in that space were most influential. Additionally, components of the practice-based methods coursework including intentional observations, reflections, peer video groups, and supervisor conferencing and coaching supported Julianna's facilitation of MP3. The data also provided evidence of the support the school district math curriculum offered both Julianna and her CT in planning for engaging students in MP3.

*Julianna's CT and internship classroom***.** Julianna's CT played a major, if not the most important, role in Julianna's learning to facilitate MP3. Her CT had an established collaborative classroom community to support student learning with MP3 before Julianna started her clinical internship. Julianna commented, "we do a lot of table talk, so they know the expectations, but I also remind them of what norms we have for that" (Interview, p. 10, lines 5-8). Norms for

discussions among students and group work in table talks were instituted by Julianna's CT and were already set up for this practice (norms established, students were used to having conversations with one another about math, work together to solve problems) which play a major

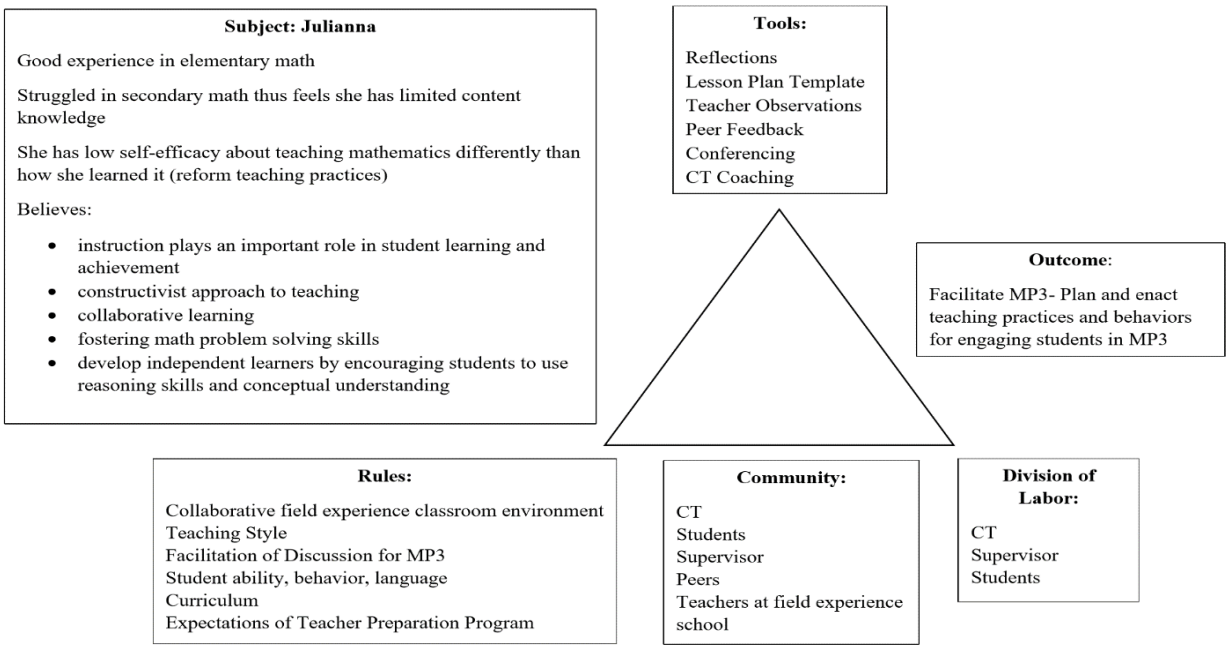

Figure 6. Julianna's Activity System for Facilitating MP3

role in supporting Julianna learn how to facilitate MP3.

Further, Julianna and her CT engaged in regular conversations regarding MP3, she indicated she asked specific questions such as, "how do you ask questions to keep everyone engaged?" (Interview, p. 16, lines 35-36). Julianna described a similar conversation,

> How do you know which students to call on, why did you call on so and so for that problem? And she said, oh well, I noticed he didn't solve this problem correctly and I walked around and I saw other people doing that and I thought it was important to point out that misconception to the whole class (Interview, p. 17, lines 10-16).

Conversations with her CT also provided her knowledge to notice, think critically about, and ask questions of her own CT with the lens of MP3. Julianna described how she would observe her CT teach as she "took notes and wrote down questions that I saw her ask. And after a while I would look at the lesson and think, hmmm, do any of these questions apply that I could start integrating into my lessons?" (Interview, p. 17, lines 1-3). This allowed Julianna to employ the same questions, copy, or mimic, what she was seeing her CT doing to her own lessons.

Julianna had a meaningful relationship with her CT. They worked closely and had consistent significant conversations focused on teaching particularly those practices related to MP3. Julianna's CT was supportive in situations for risk-free practice and facilitation of MP3 which allowed for building confidence in the classroom. Julianna and her CT used co-teaching methods and her CT was available to jump in and support her if needed. This also allowed Julianna to observe her CT engage in practices related to the facilitation of MP3 while receiving in-the-moment feedback and coaching. Julianna and her CT were active in planning lessons together, she indicated, "my CT made sure that I understood what I was teaching and that I had the materials I needed to be successful, she was very responsive when asking her questions and made sure I was confident and prepared in facilitating my lesson" (Observation Reflection, p. 2). As Julianna had more conversations and observations with her CT and in her clinical internship classroom, she developed knowledge of students and the ways that students respond, talk, have conversations, learn math, and express themselves which is needed to know when learning to respond to students.

Practiced-based methods coursework. Julianna gained more and more knowledge about MP3 as she engaged in activities in the practice-based math methods coursework. Components of the practice-based methods coursework including intentional observations, reflections, peer

video groups, and supervisor conferencing and coaching supported Julianna's facilitation of MP3.

Intentional observations. Using Bostic, Matney, and Sondergeld (2017) protocol for intentional observation helped Julianna as a novice teacher to know what to look for during teacher observations. She noted in her reflection of teaching observations that a "teacher asked the student if they agreed or disagreed with their classmate, but also asked for the student to elaborate on why they agree or disagree" (Reflection of Field Observation 2, lines 5-7) and that she was planning to use this strategy in her own math lessons. I further observed Julianna use this agree/disagree strategy frequently in her math classroom throughout her clinical internships.

Several observations across grade levels allowed for Julianna to gather knowledge about MP3 and provide a variety of examples of how to facilitate MP3. In looking at Julianna's reflections of field observations, there were many connections to what she saw during these observations and how she facilitated MP3, including "the teacher asked students if they agreed or disagreed with their classmate and asked for the student to elaborate on why they agree or disagree" (Reflection of Field Observation 2).

Reflection. Julianna's lesson observation reflections, field observation reflections, and video reflections provided insight into how she considered not only her own practice but others' teaching practice. This retrospective thought process informed her practice in noticing behavior that she was unaware of and thinking, "looking back, I should have led the lesson to be more student-centered by allowing students to talk through their strategies. I could have monitored the lesson and guided students when I noticed they were on the wrong path to finding a solution. Then my role would be to engage them in higher-level questioning to determine why they were headed in the wrong direction (Peer Video Group Reflection, p. 2, lines 1-5).

Peer video groups. Julianna felt engaging in peer video groups was beneficial and encouraged,

> a collaborative discussion around reviewing teaching practices and providing constructive feedback to make people better educators. I didn't realize that I did a majority of talking throughout my lesson until one of my peers comment on that during their review of my video. It allowed me to find my strengths as a teacher, as well as my weaknesses (Peer Video Group Reflection, p. 3, lines 12-14).

This quote also described how videos can be an important tool for reflecting on one's own teaching practice. Julianna realized how she was doing most of the talking in her lesson and then think about how to engage students in more discussion-based instruction.

Supervisor conferencing and coaching. As her Supervisor, I kept the facilitation of MP3 at the forefront during planning conferences. Having a supervisor that was also her math methods instructor provided Julianna opportunities to talk through lesson plan, address math teaching pedagogy to "ensure that I had a handle on teaching the material, how to address misconceptions, I am asking my students the right type of questions, and plan for if students aren't meeting the lesson objective" (Supervisor Observation 2 Lesson Reflection, p. 2, lines 1-9). Additionally, Julianna brought questions and concerns to be addressed in the conference to prepare for a successful math lesson and therefore "felt confident and prepared to go into her lesson" (Supervisor Observation 2 Lesson Reflection, p. 2, lines 13-16).

*Curriculum***.** The curriculum used at the school where Julianna completed her clinical internships aligned with Common Core Standards and supported the facilitation of MP3. The teacher resources for the curriculum also provided sample discussions, questioning, concept connections (debrief) and high leverage tasks supporting and promoting all Common Core

Mathematical Practices including MP3. She indicated how she used the suggested conversations in the curriculum as a guide,

> if you go through it and you look at what they want you to get at, the type of questions they ask, then you can use that when teaching the lesson. And they have a student debrief at the end of the lesson and it's a great resource (Interview, p. 19, lines 24-29).

I have included this as an influential factor because Julianna and her CT both closely adhered to the components of the curriculum and its resources to teach mathematics, including using sample discussions, questioning, and tasks.

Tensions. Tensions, or contradictions, within an activity system, can emerge at any time among the components of the system and are important for understanding interactions between key elements and how TC learns to facilitate MP3. For Julianna, tensions arose in facilitating MP3 addressing student behavior, ability, and language.

***Student behavior*.** Julianna expressed tensions when monitoring and sustaining student engagement of MP3 during small group discussions. She denoted the discussions among students that happen in math class were different than discussions that happen at other times outside of the classroom such as those conversations that happen at lunch, on the playground, or at home. Teaching students to have discussions surrounding their math ideas was challenging for Julianna despite being in a classroom where these norms were already established. She described this tension and how she addressed it in the classroom as,

> A lot of times, when they're not talking about math, they are out of their seats, or they're scooched back from their desk lounging a little bit not looking at each other where we'd say effective table talk is when you're facing that partner, you're showing them your work. So, you're looking at something, you're not just having a regular conversation.

> You're again justifying your answer to your partner by showing them what you did. So, you should be going back to your work or if it was like a reading lesson back in the text and providing proof for your answer (Interview, p. 12, lines 3-12).

Recognizing student behavior for productive math discussions is an important piece in monitoring student engagement in MP3. While teachers can't listen to more than one discussion at a time, it is critical that a teacher knows how to scan a classroom quickly for behavior that is conducive for conversations about math versus a "regular conversation".

Language and student ability levels. Perhaps the biggest tension Julianna acknowledged was attending to language and student ability levels in facilitating MP3, she described,

> Having students who are lower and some that are gifted is very challenging, especially when they're all grouped at a table and you want them to talk about their answers. Some are completely off and some got it and it's hard for them to come or have that conversation when someone has that math language and knows what they're doing compared to the other one who's intimidated. So that was something we ran into where we had to work on the tables that we had and really make sure that it was a good pair so that it one student doesn't give the right answer. You know, kind of conversation where we could really have them talk it out (Interview, p. 14, lines 14-31).

She held varying expectations for these students in attending to the math language in the lessons and translating for students who need support,

> If I know the student can, then I expect it of them. But I know for some students who aren't even on the correct reading level, that that's hard for them. Like it's hard for them to get that kind of language. So if I have them explain it to me in their own way where I can understand what they're doing, then that's acceptable. But my students who are gifted, I

> really encourage and expect them to. And that's when, during our problems set when they come over to get it checked, I'm like, explain to me how you did this or explain to me how you got that answer. And they'll say, well, I just added, I'm like, okay, what numbers did you add in? Why? I make them accountable for it because I know that they're capable of that. Whereas my lower ones, I don't need the math language. I just need you to tell me what you did and then I will reinforce some math language in that conversation (Interview, p. 13, lines 13-28).

Julianna became the translator in supporting students in hearing and using math language (described above in facilitating discussion section). She used students as models for using math language while scaffolding and reinforcing for others. It is important to mention that Julianna did not allow language to prevent students from participating in sharing their ideas with one another. She explains her method of scaffolding and supporting students,

> I try to call on them because I want them to participate. I want them to share out and they do have great ideas, but that's when I modify it. I had to modify what they say, um, for everybody to understand. And sometimes they're the only ones who participate because they're the only ones that know the answer (Interview p. 12-14, lines 16-44,1-3)

Julianna encouraged students to participate in discussions supporting MP3 and in explaining their ideas to others. She supported this by becoming a translator for math language while also asking process questions to support the verbalization of math ideas.

Case 1 summary. Julianna's initial planning for the facilitation of MP3, constructing viable arguments and critiquing the reasoning of others, included providing questions that could be generalizable to any math class and not specific to any one class or student. Over time, Julianna's lesson plans and observation pre-conferences provided evidence that she was beginning to plan

questions, not only to think deeper about the math content but also think about and respond to the work of their peers.

Julianna's facilitation of MP3 progressed from having a predetermined set of behaviors towards actions that were in-the-moment responses dependent on student behavior. Julianna provided her students opportunities to work collaboratively to share and listen to others by engaging her students in both partner and group work. Group work, such as table talks and partner work, such as turn and talks allowed students to work together on a problem to discuss different ideas and agree on a common solution.

Julianna had supported the idea that having students see multiple ways to solve problems is important for student learning. She randomly and intentionally called on students to model strategies in whole class situations. She intentionally chose students to address misconceptions and model a more effective way of solving problems. She also provided opportunities for students to see strategies with teacher modeling. Julianna expressed that providing incorrect strategies to her students allowed them to use higher-level thinking to critique and reason about why it is incorrect rather than correct.

In a whole-class instruction, Julianna's structure for facilitating discussion addressing MP3 suggested a pattern I have labeled Question-Observation-Share-Prompt (QOSP). *Q* signifies posing a question or task to the whole class launching whole-class discussion. *O* represents observation because Julianna then waits and observes action responses from students, for instance, by either show of hands or holding up personal whiteboards. In this time, Julianna also monitors students' work deliberately looking for solutions to be shared. If she notices misconceptions, she will want to address them as a whole class. Additionally, if she does not observe a certain solution, she intentionally wants to highlight then she will want to model that

herself. Next, *S* denotes share because then she invites student(s) to share their ideas and respond to the question either with peers or with the whole class. When sharing for whole-class critique, sometimes students stay at their seats and sometimes she has them come to the board to show their different strategies and explain it to their classmates for analysis and discussion. Each student that presented their ideas is subject to probing and/or prompting questions, therefore the last piece of the structure is labeled *P*. This describes when Julianna prompted or probed students by asking why or how questions to support them in explaining their solutions.

The prompting and probing questions Julianna asked during the last part of the pattern described above rely on in-the-moment thinking to decide whether she needs to ask probing questions to learn more about student thinking or prompting questions to support students in explaining or justifying their answers. They require Julianna to listen to students, make sense of their work and language, and respond/provide feedback for a deeper understanding of the math content in the lesson.

In working with students in small groups or one-on-one, Julianna used questioning to help struggling learners while also challenging others to think more deeply about the math content. She begins to help students by asking procedural questions (questions that help students with the process of math problems).

Julianna's Activity System highlights components influencing the planning and facilitation of MP3 including intentional observations, CT, internship classroom environment and classroom community, and opportunity for working with children over time. Tensions that exist within the system also emerged. Student behavior and ability levels, and language proved to be challenges for Julianna's facilitation of MP3.

Case 2: Vanessa, The Reflector

Background and teaching beliefs. Vanessa enjoyed learning mathematics throughout elementary and secondary school. It came easy to her and she was typically one of the first students in her class to grasp math concepts. She has always had positive experiences in learning math, liked her math teachers, and was successful. Learning math for her was like solving puzzles, which she loves and still does today. She particularly enjoyed upper elementary grades in learning multiplication and long division. Vanessa enjoyed completing worksheets with several math problems. Her mother would give her worksheets with math problems over the summer.

She fondly recalled happy memories doing math at home and with her teachers at school. She remembered having her third-grade teacher using race cars to show progress with learning multiplication facts. She would get bothered when she was not the first one in her class to pass the race cars. Growing up her dad would give her multiplication problems to solve in the car for fun such as 15 X 27. She said that was their bonding time and they even compete solving math puzzles now as adults. She credits her success and "natural ability" to do mathematics as the reason why she wants to teach Math and Science in upper elementary school.

According to Vanessa's Mathematics Teaching Platform, she envisioned her math class to be focused on problem-solving by providing "students a problem to manipulate that requires a deeper level of thinking, where they can actually prove how and why they got their answer" (Teaching Platform, p. 1). Vanessa also intends to have a classroom that fosters collaboration among students where "students can turn and talk about different mathematical problems as well as working in groups for hands-on activities (Teaching Platform, p. 2).

Clinical internship context. Vanessa requested and spent the first two semesters (level one and two) of clinical internships in a fifth-grade classroom. Entering her senior year, as required by the teacher preparation program, Vanessa was placed in a primary grade classroom. She spent level three clinical internship in a first-grade classroom. For final clinical internship, she requested to be placed back with her fifth grade CT. Unfortunately, a week before clinical internship started, her fifth grade CT took another position in the district. Vanessa had a great relationship with her first grade CT and made the decision to remain in the first-grade classroom. She attended and participated in first-grade planning meetings. The data for this study was collected overlapped her time in both fifth grade and first-grade clinical internship classroom, however, a majority of the data was collected during her time in first-grade clinical internship.

Codes table. The following codes table (table 6) was created after analyzing 75 pages of data from Vanessa's autobiography assignment, math teaching platform, field observation notes and reflections (3), peer video groups notes and reflection, lesson plans (2), supervisor observations notes, supervisor observation video notes, pre-conference video transcript, observation reflection, and interview transcript. I began data analysis with open coding where I read through the raw data multiple times. As I read and reread through raw data, I wrote down notes and took notations of thoughts, comments, wonderings, and connections relevant to the facilitation of MP3 in the margins nears chunks or bits of data. After working through the data in this way, I constructed and assigned color codes to my notes that were common and related together. There were seven assigned codes at this time including, Share/Listen/Read (SLR), Compare (C), Critique/Revise (CR), in-the-moment (ITM), Environment (SE), Question (Q), and activity theory (AT) (Appendix O).

After I assigned these codes to raw data, the next step, axial coding, involved grouping them as they related to one another. From the running list of the seven codes gathered (above) during the initial open coding process, I compared them from one set of data to another, this time noting comments, thoughts, and wonderings as to the emerging patterns. I used jottings (Miles, Huberman, & Saldana, 2014), drawings, and doodles to record emergent thoughts or ideas coming to mind as I reflected on the data. It is important at this point in data analysis that I became more deductive as I "tested" the category schemes to see if they held up as I further analyzed data. The process of testing codes included refining, revising, collapsing, expanding, and naming them as the data necessitated.

Quite soon into this testing process, I renamed the code for share/listen/read (SLR) to providing opportunities for students to share, listen to, and read the solutions of others (SLR). For Vanessa, these opportunities happened in both whole class and partners and included visuals to support student explanations. Further, initial codes of compare (C) and critique (CR) were collapsed into one category as they described occurrences happening simultaneously or sequentially with Vanessa exposing students to multiple strategies for solving problems.

The theme of questioning (Q) was first renamed as asking clarifying questions (Q) and then collapsed into facilitating discussion for MP3. Upon further analyzing, I noticed a pattern emerge in the way that Vanessa uses questioning when facilitating discussion.

When looking at the codes labeled with environment (E), they described Vanessa's challenges for establishing and facilitating an environment for risk-free sharing. I decided to collapse this code under the activity theory category describing tensions within the activity system. Finally, each chunk of data was then sorted into these categories as evidence preserving identifying codes and I created the final codes table (Table 6).

Looking across Vanessa's Peer Video Group Reflection, three Field Observation Reflections, Lesson Observation Reflection, and Interview Transcript, she indicated thoughtful thinking and meaningful implications for her teaching practice around facilitating MP3. Her challenges and tensions were exposed throughout her reflections of observations of teaching and enacting MP3 in her clinical internship classrooms. That is why I have titled her, The Reflector.

Table 6.
Vanessa's Data Analysis Codes

Code	Descriptors
Provide Opportunity for Students to share, listen to, or read the solutions of others (SLR)	Position students as authors of ideas who explain and justify their work (whole class and partner) Purposeful sharing Selecting and sequencing student approaches for whole class analysis Use of Representations/Visuals in explanation of solutions Teacher Modeling
Facilitating Discussion for MP3 (FD)	Whole-Class Types and patterns of questions Questioning to support struggling learners
Compare, Contrast, Critique, and Revise Solutions (C)	See and encourage the use of multiple strategies Decide if the arguments of others make sense Correct vs Incorrect Solutions Correct Solutions Teacher Modeled Solutions Teacher Intentionally Provides Incorrect Solution for students to correct
In-the-moment decision making (ITM)	Supporting struggling students Use of representations/visuals
Activity Theory (AT)	Supports CT Observation and Coaching Observations Practice Tensions Time Constraints Self-Efficacy with teaching Disconnect with teaching beliefs and CT Classroom Knowledge and Confidence teaching first-grade reform curriculum Establish and maintain an environment for engaging students in MP3

The next sections address each research question separately and describe how Vanessa facilitated MP3. The first two sections describe Vanessa's planning and enacting teaching behaviors and practices for engaging students in MP3 and each begins with a vignette to illustrate an overall picture drawing of her experiences as a result of data analysis. I used a vignette to illustrate a holistic overview of the typical conversations and observations with Vanessa. Due to my role as supervisor and math methods instructor throughout Vanessa's time in the teacher preparation program, I had broad access to her coaching and observation experiences and the vignettes allowed me to draw on these experiences to provide thick description and intentional portrayal of her facilitation of MP3.

Research question 1: How does TC plan to facilitate MP3? *Twenty minutes into our pre-conference meeting and Vanessa and I were still discussing the use of imaginary fingers with her first graders. The idea of imaginary fingers is confusing to her and is a large part of the math lessons the following week. She thinks the idea of imaginary fingers is too abstract for her students.*

"They won't understand. If we are subtracting 15 minus 8 and I ask them to show me 15 fingers they won't understand how many pretend fingers they need." I can sense the frustration in Vanessa's voice. I suggest,

"How can you reword it or scaffold the idea so it makes sense to your students? Instead of 'show me 15 fingers' can you say "show me ten fingers, if I want to show 15 how many more fingers would I need?"

Vanessa makes a joke, "I should have them take off their shoes and use their toes too".

I explain how the curriculum is trying to explicitly teach subtraction with teen numbers by using decomposition. "The decomposition of 15 into 10 and 5 and then build on students' knowledge of their addition math facts. In this lesson, it relies specifically on their knowledge of 10-8=2".

We discuss a little while longer about the language of imaginary fingers and what manipulatives and representations can be used to reinforce this idea. As we practice the language and how to present this concept to her first graders, I can see she is getting more comfortable with the math concept. I look at my watch and notice we have only 5 minutes left. I would like to explicitly discuss the ways she is planning on facilitating MP3 during this lesson. When we talk about sharing answers, I think this may be a good time to bring up MP3 and I say, "ask them 'how do you know that' this is really important. You may use thumbs up and thumbs down to have them compare their answers with their peers' answers. Pay attention to both the thumbs up and thumbs down. It's good to ask those who have a thumbs down, 'why do you disagree".

"Sometimes they just copy their neighbor, if they are giving a thumbs up" Vanessa suggests thumbs up, thumbs down isn't a good method for having her students compare their work with each other. I suggest a solution to her dilemma,

"You can have them close their eyes. And randomly pick three people why do you agree".

"Ms. Smith has them come up to the board and do it. But it takes so long. I would rather call on a kid and have them tell me. Because it takes them five minutes to come to the board with all their stuff. And it takes them forever to draw their picture."

Again, I suggest a solution, "Yes but if you even have one student come up, it provides a great opportunity for students to justify their answers, my answer is right because, and I would make it go faster by telling students that you are looking for 1 mathematician of the day. Tell them as you are walking around you are looking for 1 student who is going to be our mathematician and

is going to come up and present their solution and teach the rest of the class and explain what they did. And as you are walking around, I would intentionally be looking for a student who a: look for someone who did it correctly, or B: provide an example of a misconception. You may see many students making the same mistake, and you want to highlight that mistake. So, your students can point out what is wrong about the problem. You can have both students come up and debate their strategies or solutions. Let's listen to Natalia, let's listen to Jose and let's figure it out together."

Vanessa responds "I am just thinking I need to be much more student-centered with the little ones because to me there is so much stuff that I can do quicker than like 5th graders when they come to the board it doesn't take 10 minutes".

And with that revelation, our time had abruptly come to an end. It was a short conversation and I hope that she is at least thinking about engaging her students in ways to listen to and compare their ideas with one another. It's a start…

The first research question was designed to describe the ways TCs plan and highlight teaching practices/behaviors intended for the facilitation of MP3. The primary sources of data to answer this question spanned Math Methods Courses I and II assignments including lesson plans and planning portfolios. Additionally, level three observation pre-conferences with supervisor (myself), observation lesson plans, and interviews were included as sources. The data traversed the TC's time in the program (program context described in chapter 3) and thus provided a trajectory for their development of thinking about MP3 within both clinical internships and Math Methods coursework from level two interns to final clinical internship.

Vanessa's initial lesson plans briefly indicated how she would engage students in MP3 with later lesson planning associated with MP3 with a method of formative assessment of students'

thinking. By final clinical internship, Vanessa did not explicitly attend to MP3 in her written lesson planning only addressing it when Supervisor brought it up in pre-conference discussions.

Math methods course I and level two clinical internship. In Math Methods I Course, Vanessa's fifth-grade lesson plan demonstrated a teacher-directed approach for facilitating MP3. In her plans, Vanessa indicated that during the launch of the lesson she would,

> walkthrough the first problem with the class. Have students explain his or her thinking and which symbol they chose to represent the relationship between the two numbers. Have students think-pair-share about the next two problems after the teacher models the correct way to do this (Teaching Simulation Lesson Plan, p. 2)

During independent work time, Vanessa indicated a more student-centered approach in facilitating MP3 as she would intentionally "look for students who are using different strategies to correctly solve these problems" and select these students to explain how they solved the problem. She also specified she might "possibly make a mistake to see if students notice and explain why the mistake is wrong" (Teaching Simulation Lesson Plan, p. 2). Additionally, after independent work time she planned to gather the whole class together, "ask for volunteers to share their thinking for the problems" and "encourage discussion among the class so the class can solve these mistakes" (Teaching Simulation Lesson Plan p. 2). Vanessa also planned to ask students for other possible ways to solve the task. These are brief statements and do not describe as to how she would facilitate discussion with the class. However, she did provide a list of questions to ask in a previous section on the lesson plan directly attending to MP3 including "How do you know that both sides are equal, what can we do to change the expression to make both sides equal, how did you solve this equation, and why did you decide to solve it that way" (Teaching Simulation Lesson Plan, p. 2). There was evidence throughout the lesson plan she

intended to engage students in MP3. She indicated opportunities for exposure to multiple strategies in solving problems by having students present and explain a variety of solutions (both correct and incorrect). She planned questions for the purpose of probing students' thinking and assessing their understanding of math content.

Math methods course II and level three clinical internship. In Math Methods II course, Vanessa, in a first-grade classroom, linked MP3 with eliciting student thinking/understanding in the formative assessment section of the lesson plan. Vanessa planned questions to support students in explaining how they came to their solutions and their reasoning behind their actions by asking, "how they got their answers, what they did, ask them to draw me a picture, etc. I will ask students why they performed a certain action" (Peer Video Group Lesson Plan, p. 1). Vanessa provided limited information as to how she would engage students in sharing ideas and discussing alternative methods of solving problems. In the procedures section of her Peer Video Group Lesson Plan, she provided some details as to how she intended to have students share strategies by,

> asking students how many more bears came to play tag, allow for turn and talk to a partner to share what strategy they used. Strategies will include, counting fingers, counting on, etc. After students chat, come back together and ask them to complete the number sentence and the number bond and to circle the unknown in both (Peer Video Group Lesson Plan, p. 3)

Further, in the procedures section, she indicated she would select students to share their sentences for whole class analysis by "discussing how you can make this a subtraction sentence, write it on your board and thumbs up when you're done. Call on students to discuss options. Have students write the correct subtraction sentence on their boards as I do, remind to circle the

answer" (Peer Video Group Lesson Plan, p. 4). However, Vanessa did not specify any specific actions as to how she would facilitate these discussions or what options she wanted to address. Further in the lesson, Vanessa planned for sharing ideas by having "students participate by completing problems on their whiteboards and coming up to the board to demonstrate their work" (Peer Video Group Lesson Plan, p. 5). Again, Vanessa provided little to no details as to how coming to the board would play out during the lesson.

During a level three clinical internship pre-conference, I (Supervisor) brought attention to some behaviors for facilitating MP3 which included providing opportunities for students to compare and contrast ideas. In response to Vanessa's anticipation of students struggling with the abstractness of imaginary fingers, I suggested, "have them work in partners because you have more fingers and they don't have to set aside the imaginary numbers. Have them check their answers. Say, let's check our answers. How many fingers did we have? 10! Okay! Use visuals and then draw 10 circles" (Pre-conference Transcript, p. 7, lines 23-26). Further, I suggested she have students provide reasoning behind their answers and opportunities for students to critique and revise their ideas by "asking them how do you know that, this is really important. You can use a thumbs up and thumbs down to see if they are listening to their friends. Pay attention to both the thumbs up and thumbs down. For those that have a thumbs down, ask them why they disagree. Give them opportunities to provide reasoning for why something is right and wrong" (preconference transcript, p. 8, lines 4-7). The lesson plan did not include or indicate the intention of enacting the teaching behaviors discussed in the pre-conference. However, it should be noted here, Vanessa did enact some of the teaching behaviors discussed in pre-conference during the observation lesson.

Final clinical internship. Perhaps offering limited details regarding planning for the facilitation of MP3 during her final clinical internship was a result of teaching behaviors becoming less conscious with more practice and as facilitation of MP3 became more dependent on Vanessa's in-the-moment reaction to different students in each lesson. She explained, how engaging students in MP3 "just depends on what kid it is and how I'm going to talk to that kid to get them to, you know, engage with their peers. I don't intentionally plan for which students" (Interview, p. 7, lines 30-33). In addition, Vanessa indicated engagement in MP3 was one way she supported struggling students for "when the kids who aren't working out the problems just look like they have no idea what they're doing, so then I walk through step by step as a class (Interview, p. 3, lines 37-39). She explained how she had one student support another student,

> Once I had a kid come up to the board and do their work on the board and then they forgot how to do apart and I had another kid come up and tell them what they did and how to fix it for them in a different color so we could see what they did. Then I checked back in with the original student and made sure he understood what was happening (Interview, p. 4, lines 1-7)

Another way Vanessa used MP3 to support students in-the-moment was encouraging students to use representations to help them explain their ideas. She explains,

> as I knew my students already use imaginary fingers, I found it easier to continue with this and add the 5-group drawings as a visual to go along with it. This change was a spur of the moment change that I based off of the students' responses to my questioning (Supervisor 2 Observation Reflection, p. 1)

This quote also provided evidence of her dependence on student responses to questions asked during the lesson and how to support struggling students. Vanessa anticipated students not

understanding the abstractness of imaginary fingers in the lesson as depicted in the vignette. She planned for ways to support students with visuals if needed.

Research question 2: How does the TC facilitate MP3? *I walk in and the students are on the carpet with their attention on Vanessa at the front of the room. She reads a math problem posted on the board, "Julie rolls 8 cars down a ramp. If she started with 15 cars at the top of the ramp, how many cars does she still have left to roll down the ramp? She reads the problem again and talks briefly about the problem and reminding them of yesterday's work in math. "Work with your partners and show 15 fingers," Vanessa asks. She watches students closely monitoring their work. "Okay, how many fingers do we take away from our 15 imaginary fingers?", she asks. One student shouts out,*

"eight!"

"Okay, with your partner, take away eight fingers from your 15 fingers". She pauses to monitor and shoots a quick glance to her CT in the back of the classroom. Her CT shakes her head, yes, giving her the acknowledgment that she is doing okay. "How many fingers do you and your partner have left?" Several students call out at the same time and many with different answers. She then walks the class through taking away eight fingers.

Next, Vanessa asks her students to draw a picture, a number bond, and write a number sentence to represent their solutions. They work on their personal whiteboards and after a while, Vanessa brings the class back together as a whole group.

"Okay, who can show me what a 5-group drawing for this problem is on the board?" Vanessa chooses a student who goes up to the board and draws a picture. Vanessa notices a mistake in the student's work. She directly talks to the student and her line of questioning prompts the

student to explain his picture and the steps he did to solve the problem to the whole class. She asks, "How many circles do you have there?

"15"

"What do you do?" The student counts 10 of his circles, "What are you going to put around there? What are you going to do with that ten in the picture?" The student draws a circle around the group of ten and draws a line through another eight circles. "Why did you draw a line through those eight circles?"

"Because that is how many we are taking away"

"Why are we taking eight away?"

"That is how many cars are already rolled down the ramp"

"Okay! So, what about these circles you have left?"

"That's the ones we have"

"Right, how many are there?" The student counts,

"One, two, three, four, five, six, seven. Seven" the student looks at Vanessa to signify that this was his answer.

Vanessa dismisses the student back to their seat. She then chooses two students to come up and share their ideas on the board. She watches as they draw their pictures and number bonds. When they both finished, she asks them to explain the other's representation. Vanessa asks, "Adam, can you explain what Natalia drew here?" Adam looks at her work and explains, "she drew 15 circles, she put a tens frame around ten of them, then she crossed off eight" Vanessa asks Adam why she crossed of eight. He responds,

"that is how many cars rolled down the ramp". Vanessa responds,

"awesome!" and turns her attention back to the class and asks, "So, who can tell me what she was left with?" After a quick pause, she chooses a student with their hand in the air.

"Seven!" the student responds excitedly.

"Great!" She then turns her attention to Natalia the other student at the board and asks her to explain Adam's number bond. As she points to the number bond, Natalia says,

"His number bond is 15 in the whole and 8 is one of the parts and 7 is the other part".

"Good!" Vanessa says and turns to the class and asks, "who can tell me what a good number sentence would be? How many cars did she have left at the top of the ramp? Vanessa chooses a student, "Eloise can you go tell me on the board what number sentence helped you solve this problem?"

As the student walks up to the board, I heard Vanessa's CT address the class from the back of the room, "Watch Eloise and see what she put on her board". I can tell she wants the class to compare what Eloise is putting on the board to their own work. The student writes 15-8=7 on the board and turns toward Vanessa.

Vanessa asks Eloise, "Why did that number sentence help you solve this problem?" The student did not say anything for a few moments and then quietly whispered,

"I don't know". Turning to the class now, Vanessa asks,

"Anyone else? Why would this sentence help you solve for the answer?" Vanessa chooses another student. The student responds,

"because if there are 15 imaginary fingers and we took eight away we would have seven left".

"Are you telling me that you solved for an unknown part?"

"Yes"

"So, how many cars did Julie have left at the top of the ramp?"

"Seven cars"

"Did everyone hear that? Juan said there were seven cars left."

This question was designed to describe the ways TCs engage in teaching practices and behaviors for facilitating MP3. The primary sources of data spanned the Math Methods II course and level three and final internship clinical internships including supervisor observations, TC observation reflections, peer video group notes and reflections, and interviews.

Opportunities for students to share, listen to and read the ideas of others. Vanessa provided her students opportunities to see and encourage the use of multiple strategies for both whole class analysis and in sharing with partners. She provided opportunities for her students to be positioned as creators of ideas and solutions who share their work with others.

***Whole class*.** Vanessa routinely asked students to go up to the board to "show their work or share how they got their answers". During her observation lesson, she asked, "Jasmine, can you read your number sentence? and Can you explain to the class what you did?" (Teaching Video Notes, p. 1). During whole class analysis, Vanessa had students explain others' ideas and answer questions about the work of others such as, "Why did she cross off eight?" "what number bond did he write?" "Natalia, can you explain Adam's number bond?" (Observation Video Notes).

***Partner*.** Vanessa frequently asked students to work with a partner and sometimes referred to these pairings as Turn and Talks for students to share and listen to ideas. She described Turn and talks as a time for students to show, share, listen, and look at others' answers and does not include a discussion to explain, respond to, defend, or justify their solutions. In her first-grade classroom she described, "I think when we do a turn and talk it's more of, I'm going to tell you the answer as opposed to talking about why that's the answer" (Interview, p. 7, lines 5-7). She

reflected on the value of partner work and how talking with one another attended to justifying and reasoning about their answers,

> I don't think that I gave all the students a chance to decide if their answers were right or wrong. One thing that I need to incorporate a lot more into my mathematical teaching practice is a chance to talk to partners and discuss their work. I do not provide students with this opportunity nearly enough. I also noticed while teaching and reflecting on my video that the same couple of students who 'get it' are the only ones who volunteer to answer questions and are primarily the only ones that get called on. To alleviate this problem, I would put a popsicle stick system in place in which I pull students' names at random. But overall, I definitely need to provide all students with more opportunities to decide if their answers are right or wrong and explain why (Peer Video Reflection, p. 3)

Vanessa's reflection provided her with insight into how she enacted teaching behaviors and, in turn, was able to make improvements in her practice moving forward.

***Using visuals to support student explanations*.** Vanessa used visuals and drawings with her first-graders during sharing time as a way to support learning for different ability level students. She justified the use of visuals that supported her knowledge of student learning by stating, "As this was the first lesson on subtraction in first grade, I also think that it was beneficial for students to visually see the counters and to see a smaller group taken away from a larger group. This shows the actual subtraction action" (Peer Video Group Reflection, p. 1). She used visuals and intentional sequencing of student approaches to provide opportunities for varying levels of ability learners to share in whole group. She did this, "when we are comparing strategies if there was a harder strategy that not many kids used, I would have my higher-level kids explain that strategy, but I would maybe have one of the struggling students help by drawing a picture"

(Interview, p. 4, lines 14-19). Comparing strategies was one way Vanessa supported struggling students and provided them the opportunity to see either a correct way or a more effective way to solve a problem.

Comparing, contrasting, critiquing, and revising solutions. In her peer video reflection, Vanessa recognized the value of providing students with comparison experiences for considering other possibilities for deeper meaning and reasoning of math concepts,

> I believe that when students listen to other students' justification and reasoning it can provide additional insights for students to make about the mathematical process that they otherwise would not have made. I also think a large amount of learning can occur when students have two different answers and students think aloud to compare their reasoning and come to a conclusion of who is correct and why *they* are correct (Peer Video Reflection, p. 2)

This quote highlighted Vanessa's use of comparison experiences in attending to her fifth-graders' development of math content through justifying and reasoning about others' solutions.

***See and encourage the use of multiple strategies*.** Vanessa provided an opportunity for students to compare and contrast solutions both with teacher modeling and partner work. Vanessa provided opportunities for her students to compare correct solutions with varying strategies as well as solutions with both correct and incorrect strategies. In providing opportunities for students to compare ideas, Vanessa prompted her students to make sense of different strategies while reinforcing the understanding that there could be many ways to get a correct solution. Vanessa described one specific example of how she provided her first-graders an opportunity to see and compare different strategies,

> There was this one lesson with four different strategies and it was towards the end of the module. We were reviewing three strategies before the test. I had partners A, B, and C and they would switch off strategies for every problem. So, every kid would be able to use a particular strategy. One group would go up to the board and we would talk about how even though you all have different strategies; you all have the same answer. And why is that the case? And does it actually matter which one you use? (Interview, p. 9, lines 10-20)

Vanessa repeatedly supported and used teacher modeling to support the understanding of many strategies in explaining the efficiency of methods with students,

> When we were doing regrouping of 10, we talked about counting by tens, counting by 15, bundling to make ten, and other ways that they used. We talked about what was the most efficient method. We talk about the efficiency of them and I'll demonstrate and they demonstrate and we figure out which one is the most efficient. (Interview, p. 9, lines 26-33).

Vanessa admitted asking, "Why do we get the same answer using eight different strategies and we can do that as a class" (Interview, p. 8, lines 30-31) with first-graders needs to be scaffolded. She explains how she addressed this,

> What I have been doing is picking a kid that uses one way and a kid that uses another way and it's getting them to realize, well they got the same answer. Can they use different ways? And that, you know is the same thing. They're just showing their thinking in different ways. Sometimes I'll pick a kid who has the right answer and the wrong answer and then we will talk about what the kid did right and what he did and how we can fix the other one (Interview, p. 1, lines 37-44)

Additionally, in providing her students the opportunity to see and encourage the use of multiple strategies, she asked a recurring question for comparing solutions during whole class share time, "Why do we get the same answer using different strategies?" (Interview, p. 8, lines 30-31).

Critique and revise. Vanessa thought about first-graders purposefully revising their answers after comparing ideas, she indicated, "I noticed students like correcting their answer, but I think they are copying. They realize they are wrong and fix it and don't really think about or understand why their answer was wrong" (Interview, p. 10, lines 27-30). In providing her students with the opportunity to critique and revise, Vanessa intentionally made mistakes in her work for students to catch. She described one lesson on bundling ten where she noticed,

> I was hammering them to circle and I thought, are they just circling to circle? I saw some kids weren't counting ten before they circled and they circled all of them. So, I purposely circled 12 ones and I waited for them to notice. Some did and I asked them why they were looking at me like that. They said 'that's not ten what are you doing?' and then we had a discussion of circling groups of 10 (Interview, p. 4, lines 31-41).

This quote provided evidence of Vanessa's ability to make in-the-moment decisions for enacting teaching behaviors to engage students in MP3 while also providing her students opportunities for teacher modeled non-examples for students to critique and revise.

Facilitating discussion for MP3. Creating an environment to support students sharing, explaining, and comparing their solutions and monitoring student behavior during these discussions became influencing factors for Vanessa. Throughout her first-grade clinical internships, Vanessa indicated her tensions with establishing and maintaining an environment for risk-free sharing and supporting students to be comfortable in making mistakes. Further, she

indicated how she recognized students' body language during small group and partner conversations for monitoring student engagement in MP3.

Environment for risk-free sharing. Having students present both correct and incorrect solutions established a risk-free sharing environment and encouraged the mindset that mistakes are learning opportunities. Working with first-graders, Vanessa expressed the need for students to feel comfortable and confident when sharing their ideas with others, particularly for whole group analysis. She indicated the importance for students to feel successful in math class and supporting struggling learners by "leading and guiding them. I do not want to crush their confidence but they can still feel successful (Interview, p. 5, lines 15-24).

Additionally, Vanessa acknowledged knowing which students have the confidence to be placed in a sharing situation while having their work be critiqued was important for students to feel safe sharing in the classroom. She indicated, "I can question them even if it's about being wrong. Even if their answer is completely correct, I can ask, 'well why didn't you do it this way?' and they can defend their answers a little bit" (Interview, p. 6, lines 1-6). This quote provided evidence of how Vanessa saw the importance of students' roles in facilitating discussion around MP3. Knowing which students can share, explain, and be pushed influenced Vanessa supporting an environment where students felt safe to share their ideas.

Monitoring student engagement in MP3. Facilitating an environment for discussion involved monitoring students and be able to recognize body language and behavior conducive to talking about math. Vanessa "can tell by their body language whether or not they are talking about what they should be talking about" (Interview, p. 11, lines 11-13). She explained,

> They are normally not facing their whiteboards they are rocking on the floor, not sitting correctly, playing with something. I've noticed that some students will point to their

work, show their work, hold up their personal whiteboard, face each other, and talking (Interview, p. 11, lines 16-24)

This quote provided evidence of Vanessa's ability to scan the classroom for student behaviors while monitoring a small group or partner discussion.

Structure for facilitating MP3 during whole-class discussion. Typically, during whole-class instruction, Vanessa's structure for facilitating discussion for engagement of MP3 suggested a pattern I have labeled Question-Work/Partner-Share-Scaffold (QW/PSS). Each portion of the discussion structure included varying types of questions to support students and is illustrated in Table 7.

*Question***.** She first began by asking a question (Q) or posing a math task. For example, "What addition sentence helped you solve 15-8?" (Observation Video Notes). Math tasks presented during this time are contextual or story problems such as the cars rolling down the ramp illustrated in the vignette.

*Work/partner***.** Next, Vanessa either had students work (W) independently on personal whiteboards and/or asked students to share their ideas with a partner (P). Sometimes she labeled this partner work as a Turn and Talk. She typically prompted students by asking them if they can explain their solutions to a partner, for example, asking, "Can you explain/describe what your partner did?" (Observation Video Notes). On occasion, she may not have had students work independently, rather asked students to work with their partners to come up with a solution together, for example, "I want you to work with your partner and show me 15 fingers and take away 8 all at once" (Observation Video Notes).

Table 7
Vanessa's Pattern Structure for Discussion in Facilitating MP3 (QW/PSS)

Segment	Description	Example Evidence
Question (Q)	questions or contextual math tasks	"What addition sentence helped you solve 15-8?" "Julie rolls 8 cars down a ramp. If she started with 15 cars at the top of the ramp, how many cars does she still have left to roll down the ramp?"
Work/Partner (W/P)	Students work either independently or with a partner to answer question/math task	Turn and Talks "Can you explain/describe what your partner did?" "I want you to work with your partner and show me 15 fingers and take away eight all at once"
Share with whole class (S)	Student provided opportunity to share math ideas for whole-class analysis	"Can you explain to the class what you did?" "Can you explain to the class what your partner did?"
Scaffold (S)	Scaffolds student responses by asking probing questions	"Where did you get the two from?" Re-voicing techniques or gives a play-by-play account of what student is doing on the board "What did you do first?"

Share. After students have had time to work on their solutions, Vanessa invited students to share (S) their ideas with the whole class. She used both random and intentional methods for choosing students to present their solutions to the whole class. Questions for prompting students to share their ideas included, "can you explain to the class what you did?" and "can you explain/describe what your partner did?" (Observation Video Notes). Having students explain

another's' solution, is a one way Vanessa revealed students' deeper understanding, "those that have students explain why they think a certain way and to explain and justify their thinking or another's thinking" (Supervisor 2 Observation Reflection, p. 2) such as "Why did Natalia cross of 8? Or Natalia, explain Adam's number bond. Adam, can you describe what Natalia did?" (Supervisor Observation Video Notes).

Scaffolds. After students share their work, she then scaffolded (S) student responses by asking probing questions to support students to reveal a deeper understanding of how they got their answers. Vanessa also used re-voicing techniques or gave a play-by-play account of what the student was doing on the board. Vanessa explained, "generally, after a student's initial answer, I would ask additional probing questions to clarify their thinking and to help me understand where the student's thought process is" (Supervisor 2 Observation Reflection, p. 3). During this time, she focused on procedure questions such as, "What did you do first?" and repeated what the student said, "she drew 15 circles, then she put a ten frame around ten of them and then she crossed off eight" (Observation Video Notes). Further, she asked procedural questions to reveal student thinking such as, "Where did you get the two from? So how many do you have altogether?" (Teaching Video Notes, p. 1). Vanessa used process questions (questions pertaining to the procedural methods for solving problems) specifically when supporting struggling students, "Let's start at the beginning. Where did you put this number or what is this and point to something specific? If I can have them kind of explain their thinking and why they are doing what they are doing and if they have any idea what to do" (Interview, p. 8, lines 10-15)

At times, Vanessa asked students to justify and defend their thinking, "I try to always question why they performed a task a specific way, or how they knew this was the correct answer. This way, students are able to demonstrate a true understanding of the objective being

taught as opposed to just regurgitating an answer" (Peer Video Group Reflection, p. 1). Additionally, she asked questions such as, "why did you use the number 5? Where did you get that two from? Why did that addition sentence help you solve the problem?" (Observation Video Notes).

During this sharing time, Vanessa asked questions to support students to defend ideas, "I think it's more why questions, sometimes they will tell me something and I'll ask why? Why does that work?" (Interview, p. 10, lines 14-18). Vanessa admitted, "I tend to ask a lot of "why" and "How" questions" such as how many real fingers are we holding up? Why did that addition sentence help you solve this problem? Why am I using a ten frame? (Supervisor 2 Observation Reflection, p.2).

Research question 3: In what ways do TCs perceive supports and tensions within their activity system? Learning that happens in an activity system is dependent on several components and supports achieving certain goals in collaboration with others. In looking at the factors influencing facilitating MP3, some components of practice-based learning came into focus. Viewing Vanessa's facilitation of MP3 through an Activity System (figure 7) lens highlights components that support the planning and facilitation of MP3 including intentional observations, CT observation and coaching, and opportunity for working with children over time. Tensions that existed and that were barriers to MP3 within the system also emerged. Time constraints, self-efficacy with teaching, disconnect between teaching beliefs and CT classroom, and knowledge of math reform pedagogy proved to be challenges for Vanessa's facilitation of MP3.

Supports. Vanessa's facilitation of MP3 was supported by observations of teaching, coaching, and practice. Particularly, Vanessa's relationship with her first-grade CT played a positive role for significant coaching and observation. There were also indicators that observation of teaching of peers and other teachers were effective in supporting Vanessa's facilitation of MP3.

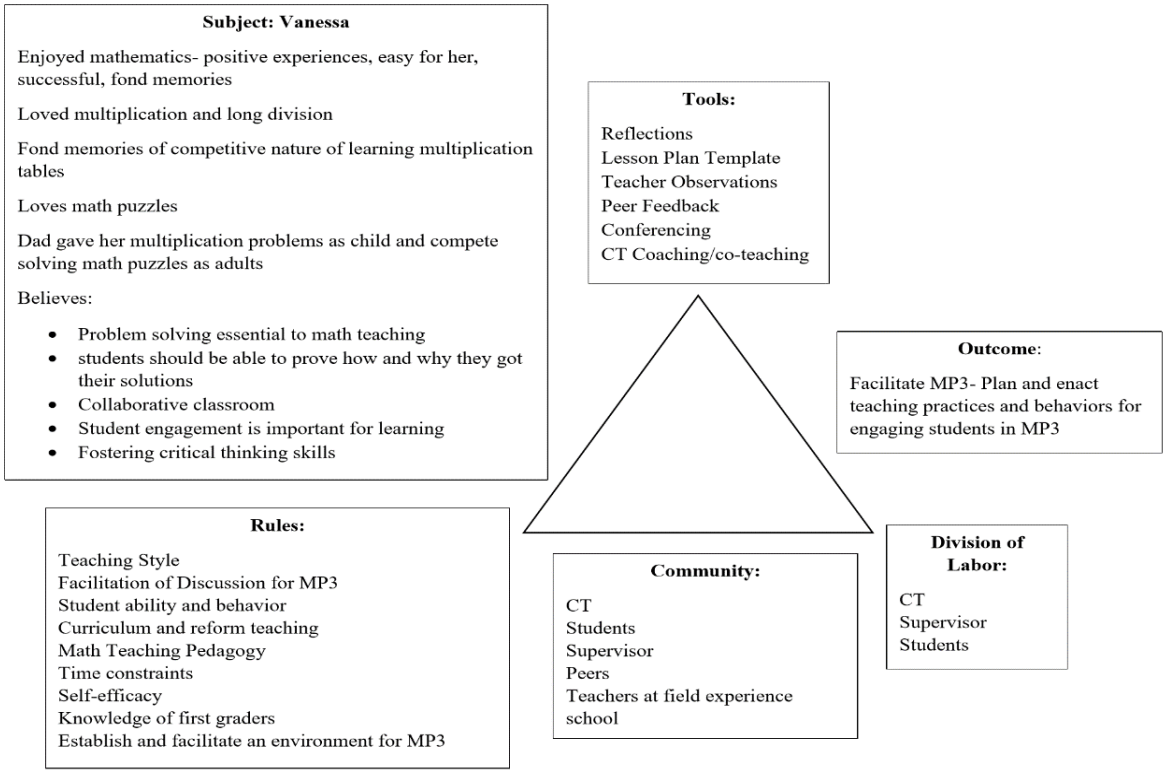

Figure 7. Vanessa's Activity System for Facilitating MP3

CT observation. Vanessa perceived her CT as a model for facilitating MP3. Observing her CT and noticing teacher behavior for facilitating MP3 influenced Vanessa's practice,

> Watching my CT in the beginning of the semester and noticing that she would put kids up there who didn't, who may have had the foundation right or they may have done

> something different way than what they were supposed to do, I liked seeing how she facilitated those conversations (Interview, p. 3, lines 11-15)

Vanessa also specified using a co-teach model provided a scaffolded method for learning to teach alongside her CT,

> When I started teaching math, my CT would pick kids and have them go up to the board and kind of lead a discussion even though I was technically the one teaching math. But it was good to see how she led the conversation, which helped a lot. For me, it was the more I was in front of the kids the more confident I got. Once I felt like I knew what I was doing, it got easier to facilitate those discussions (Interview, p. 12, lines 3-11).

Additionally, Vanessa indicated that her CT afforded her opportunities to discuss and observe her teach math provided specific teaching behavior as an example for working with children,

> My CT let me watch her teach the prior lesson so that I could observe what kind of scaffolding that she used as well as how the students responded to the concept of imaginary fingers. I also discussed the lesson multiple times and she answered all of the questions that I had. In addition, she also asked me questions about how I was going to facilitate the lesson and different consideration ideas that she thought I should consider (Supervisor 2 Observation Reflection, p. 2).

These quotes provided evidence of the powerful impact representations of math teaching influenced Vanessa's enactment of teaching behaviors and practices in engaging students in MP3. Observing others teaching mathematics provided Vanessa the ability to copy behaviors during her rehearsals of enaction. Further, discussions around observations of facilitation provided Vanessa to learn more about these behaviors.

CT coaching. Vanessa's CT provided in-the-moment support and feedback, "sometimes I'll look over at my CT when I am in the middle of teaching a lesson because I don't understand what the students are putting on the board, so I rely on her assistance of what to do and say during the lesson (Interview, p. 7, lines 35-38). The live feedback and assistance provided by Vanessa's CT allowed her to adjust her behaviors immediately during a lesson.

Intentional observation of other teachers. Using intentional observations and reflections with Bostic, Matney, and Sondergeld (2017) Look-Fors Tool played an important role in having Vanessa notice and compare what she knew about facilitating MP3 with the many ways that it plays out in classrooms. Vanessa noted what she took away from one field observation experience as she reflected on her knowledge of facilitating MP3 and what she was seeing in field observation classrooms,

> To engage students in MP3, I really like the idea of providing students with the opportunity to engage in problem-based mathematics. I think that students would be able to learn and understand more complex mathematics and gain a deeper appreciation of the problem if they have to figure it out themselves. While I did not observe any problem-based learning within this math lesson, during the application problem the CT allowed the students to solve the problem their own way as she walked around. I liked the fact that she didn't tell them which way to solve the problem; rather she allowed them to use the method that made the most sense to them. When going over the problem, she asked students to explain why they solved the way they did and asked if there were other ways to solve the problem. Unfortunately, she only showed one way to solve the problem, while certain students may have found it beneficial to see both ways. However, I really

> liked this practice and could see myself using this in my classroom (Reflection of Field Observation 3, p. 1, lines 28-37)

Again, these quotes provided evidence to support the powerful impact representations of teaching have on TCs. More importantly, these observations were intentional and were accompanied by protocols to help TCs notice teaching behaviors and practices for engaging students in MP3. Vanessa was able to see a variety of grade-level teachers facilitate MP3 and then reflect on what she saw and how those representations impacted her own teaching.

***Peer observation*.** There was also evidence that peers provided an opportunity for Vanessa to see a variety of methods for facilitating MP3, while also providing a critique of Vanessa's behaviors of engaging students in MP3,

> It was interesting having our peers critique our teaching practices and provide suggestions that I would not have thought of. For instance, Alison stated that I could more consistent in asking students to explain their thinking verbally and then asking who solved it similarly or differently if she wanted to focus more on expanding her students' thinking this way. Honestly, I had not thought about asking who had solved it similarly and that is an absolutely fantastic idea. And I'm glad that she suggested it (Peer Video Group Reflection, p. 4)

This quote provided further evidence of the influence teaching representations and feedback, specifically from peers in her cohort and math methods courses, had on Vanessa's facilitation of MP3.

Tensions. Many tensions emerged for Vanessa addressing the facilitation of MP3 including, time constraints, self-efficacy in teaching, knowledge of curriculum, and disconnects between teaching beliefs and CT classroom and practice.

Time constraints. Vanessa showed signs of frustration when talking about the time it takes her first-graders to present their ideas visually on the board and explain them for whole-class analysis, "it takes so long. I would rather call on a kid and have them tell me their solution because it takes them five minutes to come to the board with all their stuff. And it takes them forever to draw their picture" (Pre-Conference Transcript, p. 8, lines 16-18). Later during her final clinical internship, Vanessa was struggling with classroom management addressing student engagement and keeping students on-task during math lessons. Again, she described how factors influencing math lessons prevented students from facilitating discussions for MP3,

> Sometimes its very time consuming and if the kids are off that day and we are running late, it's easier just to get through the problems then to facilitate those discussions. Especially because I think at the younger grades its harder to keep kids engaged if we're just talking about it as opposed to actually writing on our boards or something like that (Interview, p. 12, lines 24-30).

This quote highlights how students and time constraints can interact in the system as rules because they limit, guide, and regulate the ways Vanessa enacted behaviors for engaging students in MP3.

Knowledge of first-graders and reform curriculum. Vanessa struggled to be in a first-grade classroom. She felt more comfortable and preferred intermediate classrooms. However, due to unforeseen circumstances, she was not able to go back to a fifth-grade classroom for final clinical internship. She struggled with understanding conceptual math concepts and reform teaching methods at the first-grade level. She expressed, "I think first-grade math is hard to teach because it is so easy for me. The students don't understand it yet. The conceptual knowledge the

curriculum is focused on I find ridiculous sometimes. I know it's important, but sometimes I do not get it" (Interview, p. 17, lines 20-28). In her observation reflection, she acknowledged,

> The biggest takeaway that I took from this observation cycle was the amount of preplanning and preparation needed for more complex and cognitively demanding tasks on my end. Don't get me wrong, I always prepare for my lessons, but this one took me a couple of weeks of mental preparation in addition to understanding the concept, I had to get my mind wrapped around this topic. I think another big takeaway from this observation lesson was having to teach something that I didn't really want to teach. I didn't care for this lesson because I didn't understand how this concept would build conceptual knowledge. But overall, I think that this experience was incredibly beneficial for me (Supervisor Observation 2 Reflection Questions, p. 4)

Vanessa expressed that part of her lack of knowledge with first-grade concepts was in part because she did not engage in these methods as a student, "I do not remember how I learned first-grade math concepts, but I didn't learn it the way I had to teach it this year. It was hard to learn and teach methods at the same time" (Interview, p. 17, lines 39-44). Having limited math knowledge about first-grade concepts was a tension for Vanessa throughout her time in her first-grade clinical internship classroom. As depicted in the vignette, Vanessa and I spent quite a bit of time discussing the math content addressed in first grade.

Disconnect between traditional and reform teaching methods. Vanessa's 5th grade CT utilized teaching methods more aligned with teacher-centered approaches. She began to notice a disconnect from what she learned in the university classroom as to how MP3 might look and what she was observing in her CTs classroom. She reflected,

> As I am with this CT every week, the MP3 that I saw today is what I see every week. However, I think that there are many more opportunities to incorporate MP3 than she actually does within her lessons. With that being said, in fifth-grade MP3 involves a lot of questions on the procedural aspects of problem-solving (that's at least what I am seeing). When I am teaching a math lesson, I like to ask the kiddos 'why' they got the answers they did. I also like to have the students explain how they solved the problem and ask students who solved the problem differently and demonstrate that process (Reflection of Field Observation 3, p. 1, lines 19-25).

However, during the facilitation of MP3, she seemed to slip into the more traditional teacher-centered approach rather than enacting the core practices she learned in the university classroom. Vanessa's teaching methods lean towards more traditional and teacher-centered,

> I have realized over the course of this year, that in primary, I tend to be much more teacher-centered than student-centered. I think that this was pretty obvious over the course of this lesson as well. For the most part, I stood at the board and taught this math lesson, while students performed problems on their whiteboards. I only provided a couple of opportunities for turning and talking, when many more could have been provided. I want the students to be truly engaged in the lesson, I need to let them play a more active role in their learning (Peer Video Group Reflection, p. 1)

As she spent more time in her first-grade clinical internship classroom, Vanessa began to learn about the conceptual math concepts,

> One thing that I learned about my students' mathematical thinking is that addition is incredibly helpful for the students to understand subtraction. I thought this was very surprising. I know they are related operations, but I didn't realize how easy it would be

> for them to subtract using addition, by trying to find a mystery number. I just feel that it is kind of a stretch in my mind, that it is so interesting to me that it helped my kiddos so much (Peer Video Group Reflection, p. 3)

The disconnect between traditional and reform teaching Vanessa experienced allowed her to explore the connections between theory and practice, or more specifically, what she was learning in the university classroom versus what she was perceiving in her clinical internships (both in observing and teaching). This brought up a rather big tension within Vanessa as she was trying to solidify her own beliefs about teaching mathematics.

Disconnect between CT practice and other teachers' practices. Vanessa expressed the differences she noticed between what her CT does in her clinical internship classroom and what she sees in observing other teachers. Vanessa suggested that she and her CT did not specifically talk about facilitating MP3. When I had asked Vanessa if her CT facilitates MP3, she responded, "I don't think she realized she is facilitating MP3, she never called it that" (Interview, p. 14, lines 16-17). In her Field Observation Reflection, Vanessa compared what she noticed in another teacher's classroom to her CT's facilitation of MP3 and noted how they are different,

> I really loved how often the teacher engaged with the students by asking what answers they all got. Because all of the students got different answers. I loved how the CT constantly asked the students if the answers that the other students were okay, and if they were, why were they okay. I think that this is a fantastic way to engage students in MP3. I have noticed this practice in my classroom, for example when students are asked to create their own equations to gage understanding. However, I feel that my CT shuts the students down too quickly and only allows one student to share their answer. I would love to make

this aspect of MP#3 more prevalent in my classroom (Reflection of Field Observation 2, p. 2, lines 1-7)

Vanessa compared how she perceived facilitating of MP3 in fifth grade versus in a first-grade classroom,

> In the higher grades, there is a foundation of when we talk, we are going to talk about this and you are going to tell me why as opposed to the little ones, they are just learning to tell, to tell someone else their ideas, arguing about them. I just think it looks different, in fifth grade it's more of defending answers and in first grade, it is more tell answers (Interview, p. 12, lines 3-11)

This quote emphasized Vanessa's ideas about how the facilitation of MP3 was different across grade levels. It was evidenced here she believed fifth-graders are more capable of justifying their answers and have more advanced conventions of conversation to engage in discussions, or debates, related to defending solutions.

***Establish and facilitate an environment for risk-free sharing and discussion*.** Vanessa demonstrated tensions in establishing and maintaining an environment that supports the facilitation of MP3. She discussed promoting a classroom climate where students feel comfortable to make mistakes, "Getting them to see what's wrong and how we can fix it sometimes I feel like it makes the kids a little uncomfortable critiquing somebody else or the kids being critiqued. But I think its if you established like that climate then I think it would be okay" (Interview, p. 2, lines 5-8). She revealed she hoped students would value mistakes as opportunities for learning, "Normally, I'm just hoping that they can recognize what's wrong with a problem and that it is okay to make mistakes and lots of times it's a very simple fix" (Interview, p. 2, lines 12-15).

However, in facilitating MP3, Vanessa found establishing this climate hard because it depended on establishing trusting relationships with her students,

> I didn't feel like I had the relationship with the students, I didn't think that they felt comfortable enough with me to have me challenge what they were thinking without coming across like I was being mean to them. I think I would have felt personally attacked and I don't think I was confident my abilities enough to facilitate that discussion of how to fix it or how to change what we did. I attended to stuff we did on the board. I had the kids who don't have the right answers go up instead of the kids who had the wrong answers. I was afraid to put them up on the board (Interview, p. 2, lines 26-40)

This salient quote called attention to Vanessa's recognition of the importance building relationships had on creating and maintaining a trusting environment for discussion. Further, the quote demonstrated Vanessa's understanding of her students' needs to feel comfortable making mistakes and taking risks for engagement in MP3.

Self-efficacy with teaching. There were times where Vanessa expressed her self-efficacy with her own abilities in teaching. She expressed her lack of confidence in supporting students to verbalize defending their answers, "I don't feel confident enough in my abilities yet to lead a debate. I feel like now I can lead a discussion, not I am not quite there to respond to students defend their answers" (Interview, p. 9, lines 40-41 and p. 10, lines 1-2). Additionally, in response to supporting students with incorrect answers and helping them to understand why their answer is wrong rather than having them just copy correct answers from partners, she stated, "I think it's a personal facilitation thing, I'm not sure how to handle that yet" (Interview, p. 10, lines 32-34).

Case 2 summary. Vanessa's initial lesson plans (math methods I) briefly indicated how she would engage students in MP3, constructing viable arguments and critiquing the reasoning of

others. Later during Math II, Vanessa's lesson planning associated MP3 with a method of formative assessment of students' thinking. By final clinical internship, Vanessa did not explicitly attend to MP3 in her written lesson planning and only addressing it when Supervisor brought it up in pre-conference discussions.

Perhaps her limited details regarding planning for the facilitation of MP3 depended on Vanessa's in-the-moment reaction to different students in each lesson. She explained, how engaging students in MP3 "just depends on what kid it is and how I'm going to talk to that kid to get them to, you know, engage with their peers. I don't intentionally plan for which students" (Interview, p. 7, lines 30-33). In addition, Vanessa indicated engagement in MP3 was one way she supported struggling students for "when the kids who aren't working out the problems just look like they have no idea what they're doing, so then I walk through step by step as a class (Interview, p. 3, lines 37-39).

Vanessa provided her students opportunities to see and encourage the use of multiple strategies for both whole class analysis and in sharing with partners. She provided opportunities for her students to be positioned as creators of ideas and solutions who share their work with others. Vanessa used visuals and drawings with her first-graders during sharing time as a way to support learning for different ability level students.

Vanessa provided an opportunity for students to compare and contrast solutions both with teacher modeling and partner work. Vanessa offered opportunities for her students to compare correct solutions with varying strategies as well as solutions with both correct and incorrect strategies. In providing opportunities for students to compare ideas, Vanessa prompted her students to make sense of different strategies while reinforcing the understanding that there could be many ways to get a correct solution.

Vanessa thought about first-graders purposefully revising their answers after comparing ideas she stated "I noticed students like correcting their answer but I think they are copying. They realize they are wrong and fix it and don't really think about or understand why their answer was wrong" (Interview, p. 10, lines 27-30). In providing her students with the opportunity to critique and revise, Vanessa intentionally made mistakes in her work for students to catch. In providing her students with the opportunity to critique and revise, Vanessa intentionally made mistakes in her work for students to catch.

Having students present both correct and incorrect solutions established a risk-free sharing environment and instilled that mistakes are learning opportunities. Working with first-graders, Vanessa expressed the need for all students to feel comfortable and confident when sharing their ideas with others, particularly for whole group analysis.

Facilitating an environment for discussion involved monitoring students and be able to recognize body language and behavior conducive to talking about math. Vanessa "can tell by their body language whether or not they are talking about what they should be talking about" (Interview, p. 11, lines 11-13).

Typically, during whole-class instruction, Vanessa's structure for facilitating discussion for engagement of MP3 suggested a pattern I have labeled Question-Work/Partner-Share-Scaffold (QW/PSS). Each portion of the discussion structure included varying types of questions to support students. She first began by asking a question (Q) or posing a math task. For example, "What addition sentence helped you solve 15-8?" (Observation Video Notes). Next, Vanessa either has students work (W) independently on personal whiteboards and/or asks students to share their ideas with a partner (P). After students have had time to work on their solutions, Vanessa invited students to share (S) their ideas with the whole class**.** She then scaffolded (S)

student responses by asking probing questions to support students in revealing a deeper understanding of how they got their answers. Vanessa also used re-voicing techniques or gave a play-by-play account of what the student is doing on the board.

Vanessa's Activity System highlighted components influencing the planning and facilitation of MP3 including intentional observations, CT observation and coaching, and opportunity for working with children over time. Tensions that existed within the system also emerged. Time constraints, self-efficacy with teaching, disconnect between teaching beliefs and CT classroom, and knowledge of math reform pedagogy proved to be challenges for Vanessa's facilitation of MP3.

Case 3: Kelly, The Modeler

Background and teaching beliefs. Kelly found elementary and middle school math easy. She even competed in mathematics competitions and in 7th grade was third in the state for computation. She felt her middle school math teacher encouraged and pushed her to be better and is still in touch with her today. It wasn't until high school when math became more challenging for Kelly. She began being homeschooled and attend virtual school. She admitted she didn't enjoy it and struggled with being "self-taught". Subsequently, she went from getting As in math to Cs. In college, Kelly failed algebra twice and blamed homeschooling for a lack of content knowledge. Kelly expressed she learned better when a teacher was in front of the classroom providing examples and visuals.

According to Kelly's Math Teaching Platform, she believes math should be interactive in exploring different ways to solve contextual math problems. She supports students in solving problems in different ways where "as long as you are able to explain it and have it make logical sense, then it is right" (Math Teaching Platform, p.1). Further, Kelly intends to "create an

environment where students will be able to effectively take risks, share and defend their ideas on how they achieved the solution" (Math Teaching Platform, p, 1).

Clinical internship context. Kelly spent her first year (level one and two) clinical internship in a third-grade non-traditional co-taught classroom with two teachers and 30+ students. This class was structured around small homogeneous ability-leveled groups and had limited whole class instructional time. For the second year of clinical internship (level three and final), Kelly was placed in a first-grade traditional classroom consisting of one teacher and approximately 20 students. She had a very close relationship with her CT, often communicating on both professional and personal matters in and out of clinical internship school. Kelly attended and participated in weekly grade-level planning meetings.

Codes table. The following codes table (table 8) was created after analyzing 80 pages of data from Kelly's autobiography assignment, planning portfolio, math teaching platform, field observation notes and reflections (3), peer video groups notes and reflection, lesson plans (2), supervisor observations notes, supervisor observation video notes, pre-conference video transcript, observation reflection, and interview transcript. I began data analysis with open coding where I read through the raw data multiple times. As I read and reread through raw data, I wrote down notes and took notations of thoughts, comments, wonderings, and connections relevant to the facilitation of MP3 in the margins nears chunks or bits of data. After working through the data in this way, I constructed and assigned color codes to my notes that were common and related together. There were initially 15 assigned codes at this time including, Partner Analysis (P), Revise and Reflect on Mistakes (R), Assess Reasoning of Others (A), Student Ability (SA), Language (L), Planning (PL), Listen/Monitor Conversations (LM),

Compare (C), Facilitating Discussion (FD), Whole-Class Analysis (WC), Many Strategies (S), Questions (Q), Environment (E), and Activity Theory (AT) (Appendix O).

After I assigned the codes to the raw data, the next step, axial coding, involved grouping them as they related to one another. From the running list of codes gathered (above) during the initial open coding process, I compared codes from one set of data to another, this time noting comments, thoughts, and wonderings as to the emerging patterns. I used jottings (Miles, Huberman, & Saldana, 2014), drawings, and doodles to record emergent thoughts or ideas coming to mind as I reflected on the data. It is important at this point in data analysis that I became more deductive as I "tested" the category schemes to see if they held up as I further analyzed data. The process of testing codes included refining, revising, collapsing, expanding, and naming them as the data necessitated.

As a result of this round of data analysis, I ended up with five final categories from the 15 initial codes from above. These five categories and their descriptors are illustrated in Table 8. Finally, each chunk of data was then sorted into these categories as evidence preserving identifying codes.

The next sections address each research question separately and describe how Kelly facilitated MP3. The first two sections describe Kelly's planning and enacting teaching behaviors and practices for engaging students in MP3 and each begins with a vignette to illustrate an overall picture drawing of her experiences as a result of data analysis. I used a vignette to illustrate a holistic overview of the typical conversations and observations with Kelly. Due to my role as supervisor and math methods instructor throughout Kelly's time in the teacher preparation program, I had broad access to her coaching and observation experiences and the

vignettes allowed me to draw on these experiences to provide a thick description and intentional portrayal of her facilitation of MP3.

Table 8
Kelly's Data Analysis Codes

Code	Descriptors
Provide Opportunity for Students to share, listen to, or read the solutions of others (SLR)	See and encourage the use of multiple strategies Position students as authors of ideas who explain their work (whole class and partner) Use manipulatives, drawings, or representations to support ideas
Facilitating Whole Class Discussion in Facilitating MP3 (FD)	Establish and Maintain Norms for Discussion Pattern Structure for Whole Class Discussion Facilitating Small Group/Partner Discussion Questioning Clarifying and Probing Questions to reveal a deeper understanding of student reasoning and actions Procedural Questions for Supporting Struggling Learners Reflective Questions Types and patterns of questions
Compare, Contrast, Critique Reflect and Revise Solutions (CCRR)	Agree or Disagree with Peers (Thumbs up, Thumbs down) Correct vs Incorrect Solutions (counterexamples) Correct Solutions Teacher Modeled Solutions Teacher Intentionally Provides Incorrect Solution for students to correct Decide if others' solutions make sense
Establish, Facilitate, and Maintain Classroom Environment for Facilitating MP3 (E)	The mindset to learn from mistakes Knowing which students to be pushed and supported Encouraging all students to have a voice and share answers
Activity Theory (AT)	Supports CT Observation, Coaching/Feedback, Co-teaching, and Clinical internship Classroom Clinical internship School District Curriculum Practice-Based Methods Coursework Observations Peer Video Groups Tensions Student Behavior Teacher-directed Curriculum Time Consuming Level 3 lack of consistent observation and practice (2 days a week)

Research question 1: How does TC plan to facilitate MP3? *"So, they are using their whiteboards with the rekenrek and numbers paths?" I ask Kelly as she describes how her students will solve 13-8 with decomposing 13 into 10 and 3. I follow up, "Are you going to have them do it with a partner?"*

Kelly responds, "yes, probably turn and talk with their whiteboards and they will come up with an answer". I think that this may be a good opportunity for Kelly to engage her students in MP3 and share their answers. I keep pressing for more information about if and how she plans to facilitate this.

"then what?"

"Then we will talk about the answers they got. So, one idea my CT had was to have them put all their answers on the board using it as a model and have them explain how they got that answer. And then having them figure out for themselves whether the answer is right or wrong". Great! I think to myself; she is on the right track! I love that she is providing an opportunity for students to critique others' solutions and reason about their work. I express my thoughts and elaborate,

"this is also a way to do what I call error analysis. So, they are checking to see if they agree with others but also checking to see well I don't agree is it because they got it wrong or I got it wrong. And if it's because I got it wrong, it gives them a chance to revise their answers".

"We did that last week with a couple of lessons" Kelly replies. She is referring to her and her CT, and it really speaks to the collaboration happening in her clinical internship classroom. Kelly continues to elaborate on how her CT typically has students share their solutions and how she intends to do the same in her lesson,

"So what my CT does is they have their whiteboards and they will bring up their whiteboards and put it on the ledge of the big whiteboard, That way all the kids can see the process of how

they got it and she lets them explain to the class how they got their answers, this is what I did. And usually, somewhere along there they find out if they did something wrong or if they got it right".

I would love to hear how she is planning to facilitate class discussion around this sharing time, I ask, "navigating that conversation, anticipating students' answers and struggles and misconceptions it falls into this conversation that you have. You might have a student share that demonstrates a misconception. How are you going to field that conversation because you want to point out to students that this is an incorrect answer and you are hoping that they see for themselves that this is an incorrect answer and why? But what if they don't see this as an incorrect answer? So, being prepared for what kind of questions are you planning for this time?" Again, she talks about how she and her CT typically address this situation. She indicates specific student actions that she will be looking for during this time,

"So, last time we did it we had students that would add an extra line or when they touched the 8 on the number path they would count that dot as one of the things. So, reminding them not to touch that line and we have them draw a dot over the top and not touch the line has been helping. And if they are adding instead of subtracting or getting their signs confused or something having that discussion." We then talk about intentionally providing opportunities for students to see and work through non-examples. I look at the time and see that we only have a few minutes left. I try to sum up this part of the lesson and see what Kelly has planned for the last part of the lesson independent work,

"So, this is group work on the carpet, working with a partner, having a conversation of how one student did it. And providing opportunities for students to learn from one another. How do you complete the problem set? Together?"

"We will do the first one together and then they will go back to their desks and complete the others independently".

The first research question was designed to describe the ways TCs plan and highlight teaching practices/behaviors intended for the facilitation of MP3. The primary sources of data to answer this question spanned Math Methods Courses I and II assignments including lesson plans and planning portfolios. Additionally, level three observation pre-conferences with supervisor (myself), observation lesson plans, and interviews were included as sources. The data traversed the TC's time in the program (program context described in chapter 3) and thus provided a trajectory for their development of thinking about MP3 within both clinical internships and Math Methods coursework from level two interns to final clinical internship.

Kelly's initial lesson plans briefly indicated how she would engage students in MP3 with general behaviors attending to facilitation. Later lesson planning indicated further specific behavior for engaging students in MP3 more frequently. By final clinical internship, Kelly became more responsive to making connections to math content and attending to specific students' needs in facilitating discussion and use of math language.

Math methods I course and level two clinical internship. Math Methods I Course was taken during level two clinical internship and provided evidence Kelly knew certain behaviors for facilitating MP3, however, these behaviors were generalized without providing specific information as to how they connected to math content or student learning such as anticipating or addressing student responses. For example, Kelly planned for asking students "higher-level questions and then to defend their answers" (Planning Portfolio), but she did not indicate what higher-level questions she planned on asking. Additionally, Kelly planned to provide her students opportunities to decide if others' solutions made sense after students present their

solutions by asking "the rest of the class if they agree and disagree. Why or why not?" (Planning Portfolio). She did not indicate what she would do whether students agreed or disagreed. Further planning of generalized behavior for the facilitation of MP3 was indicated from Kelly's plan for "asking the students to look at their answer, does their answer make sense? Is there another way to solve the problem?" (Planning Portfolio).

Math methods II course and level three clinical internship. Math Methods II Course was taken during level three clinical internship and seemed to provide evidence Kelly planned for more frequent behavior for facilitating MP3 while attending to student learning. She planned to provide opportunities for students to justify their solutions by asking "individual students for the answers and then follow up their response with 'why?'. Further, Kelly planned to provide students to see and listen to solutions to others by calling "on other students to tell me other ways that they could solve the number bond" (Peer Video Group Lesson Plan). During Math II, Kelly also planned for students to work in partners for opportunities to solve problems together indicating, "students will work with their shoulder buddy to solve Jose's Cherries 8+___= 14" (Supervisor Observation 2 Lesson Plan).

Kelly indicated a plan for providing students opportunities to share and listen to multiple solutions with an opportunity to work with partners to explain, listen and critique the solutions of others, and revise solutions,

> I will ask them to share their answers and will write all of the answers given on the board (whether they are right or wrong). Once I have gotten everyone's answer I will have the students turn and talk to their shoulder partner. Each partner will have one minute to share how they came to the answer they got. I will call students back together and will go through the answers that were previously written on the board and ask if they still think

> they have the right answer or if they found a mistake and have a different answer. If they have a different answer, then I will cross off their previous answer and move on to the next. For answers that are left, we will work it out on the board and see where the student went right/wrong" (Supervisor Observation 2 Lesson Plan)

This quote provided evidence of Kelly's intention to offer opportunities for students to share their ideas for the whole class and partner analysis. It also illustrated the opportunity for students to critique and compare ideas with the possibility of revising incorrect solutions.

Final clinical internship. Kelly's planning for the facilitation of MP3 during her final clinical internship indicated being responsive to making connections to math content and attending to specific students' needs in facilitating discussion and use of math language. She specified her plans to attend to "the conversation piece and constantly using the math language as well while teaching the lesson, but also having them say it back to me and repeating it so they get comfortable with the language of it. I think it helps to have a better understanding of what they are doing" (Interview, p.3, lines 1-7).

Kelly's plans also suggested her reliability of the district math curriculum as she copied procedures from the curriculum into her lesson plan. This suggested her lack of planning to facilitate MP3 for her specific students beyond the generalizable curriculum plans such as,

> I will ask a volunteer to tell me how I would show this total on the rekenrek on the place value chart. I will stop at random points to ask students how I would show how many beads I have in my place value chart. How would I write this in my place value chart? If I were to read this number in the place value chart, what number would say I have? (110) We will continue the process until we get to 12 tens/120. (Supervisor Observation Lesson Plan Final Internship).

At the end of the final clinical internship, Kelly indicated her intentions to facilitate MP3 for future lessons and suggested engaging students in MP3 became a less conscious effort. During the interview when asked specifically about how she planned to engage students in MP3 during her lessons, Kelly responded,

> Ultimately, the goal is to incorporate it into all of our lessons. Now, whether we successfully get to that point in every single lesson, that's another story, but I would say that the goal is to incorporate a time where the students are asking those hard questions and inquiring about how did they solve this problem and critiquing their peers is a big part of it. (Interview, p.3, lines 35-38).

This quote highlighted Kelly's intention to continue to engage students in MP3 as a reoccurring experience and embedded as part of her math instruction. However, it also suggested that not all lessons may include MP3 and is dependent on other factors of the lesson.

Research question 2: How does the TC facilitate MP3? *I walk into Kelly's first-grade classroom. The lights are off, the kids are sitting cross-legged on the carpet in front of the smartboard. The students are equipped with personal whiteboards and dry erase markers. They are looking up at Kelly as she reads the math problem off the smartboard, "There are nine beads on the floor. A student picks up some beads but there was still some on the floor. Write a number bond and number sentence that would match this story". Kelly immediately begins to draw circles on the board asking students, "so, we are going to draw a model first. How many are we going to draw?"*

Some students call out, "nine!"

Kelly, re-voices the students' answer, "Nine, okay draw nine beads". She draws nine circles on the board as the students mimic her on their personal whiteboards. Kelly pauses as she waits for

the students to finish drawing their circles. She looks around at various students watching them draw their circles. After a few moments, Kelly calls on a student to give her the next piece of information, "Julia, how many do you want to pick up off the floor?" I can tell Julia is thinking. After a few seconds Kelly supports Julia by asking her to pick a number between zero and nine.

"One" Julia answers.

Kelly instructs the class, "Okay, we are going to pick up one bead. So cross off one bead because that's the one that is going to be picked up. Now we are going to draw our number bond". She starts drawing a number bond on the whiteboard next to the bead model. How many did we have to begin with on the floor?" She asks. Again, some students in the class call out nine. Kelly re-voicing the students says, "Nine" and puts a nine in the big circle in her number bond. "How many did Julia pick up?"

Some students call out, "One"

"One, yes. So, how many do we have left on the floor?" Kelly asks as she puts a one in a smaller circle in the number bond.

"Eight", a few students call out.

As she adds the eight to the last circle to complete the number bond, once again Kelly restates the students' answer, "Eight. So, now we are going to write our number sentence." Next to the number bond, she begins to write a number sentence on the board with three blanks like this, ___ - ___ = _____. She rereads the original word problem from the board filling in the information from the number bond. As points to the parts of the number bond, Kelly says, "Okay, so we have nine beads on the floor, Julia says we pick up one bead so that left eight more beads on the floor. So, now our number sentence. How many did we start with?"

"Nine" various students call out.

"Nine", Kelly re-voices the students' response as she writes the number nine in the first blank in her number sentence on the board. She then asks, "So, we have the first number in our number sentence. How many did she pick up?" Again, a few students call out,

"One."

Once again Kelly restates the student's reply and writes a one in the second blank in the number sentence. "How many do we have left on the floor?" Some students respond with eight and Kelly writes an eight in the last blank in the number sentence. She pauses and looks at the class. She waits for some to finish writing their number sentences on their whiteboards. "Does yours look like mine she asks?" After getting affirmation from the students such as shaking heads, and saying yes, she points to the number bond on the board and asks, "I have a question for you, is eight and one the only two number parts that can go into the bottom of this number bond?"

"No!" some students answer enthusiastically.

Kelly repeats the students answer, "No!, What other numbers can go into the bottom of our number bond?"

One student replies, "We can do five plus four!"

"Five plus four? Are we using plus or are we taking away?"

The student responds, "taking away."

"So what sign are we going to use?"

"The take-away sign"

"Right, the takeaway sign". Kelly then addresses the whole class as she writes in the number bond circles, "So we took five beads away this time. What is going on the bottom of our number bond, Carlos?"

Carlos responds with the correct numbers of five and four. Kelly enters them into the number bond and begins to fill out the corresponding number sentence much like before. She asks questions that highlight the step-by-step process of thinking about the whole number as the first number in the number sentence and then using the parts for the next two blanks. She continues to reinforce the context of the math problem and picking up beads and finding out how many are left on the floor. Kelly then asks the students to come up with another way to break down nine. The students work first independently and then share their work with their shoulder buddies at their desks. As she walks around, she stops at a few desks to talk to students pointing to their work. Each stop is only a moment or two. After a few minutes, she brings the class back together.

"Did anyone come up with a number set we can use?"

A student calls out, "Nine and zero, nine minus zero"

"So, you didn't pick up any beads?" Kelly asks the student.

"Nope" the student replies.

Kelly turns her attention to the class and asks, "Does that work? Talk to your shoulder buddy." She pauses for a few moments as she monitors students talking with one another. She brings the class back together by asking, "Do we agree or disagree?" Kelly looks around and sees students put thumbs up in the air. "Does anybody disagree?" Not one student disagrees. "Okay, let's find out", she says as she begins to think aloud and begins to fill out a number bond on the board, "We have nine, we picked up zero, there is still nine on the floor." She points to the first blank in the number sentence. "Which number do we put here?"

A student calls out, "Nine"

"Why does the nine go first?" she asks the student.

"Because if it was the second number, I would be picking up nine, but I am picking up zero. Nine is how many total".

"That's right, that's how many beads we have total". Kelly finishes plugging the last two numbers in the number sentence. Kelly looks towards the rest of the class and asks, "Is this what yours looks like?" She does the same pattern for choosing another students' solution of six and four. Drawing a number bond, asking the students which numbers plug into the spaces in the corresponding number sentence. Asking why each of the numbers goes in each spot and asking the class if they agree or disagree with what she has written on the board. At the end of the lesson, she reviews all of the number combos or "number families" they used during the lesson. She finally sends them off to independently complete a workbook page.

This second research question was designed to describe the ways TCs engage in teaching practices and behaviors for facilitating MP3. The primary sources of data spanned the Math Methods II course and level three and final clinical internship including supervisor observation notes, video notes, TC observation reflections, peer video group reflections, and interview.

Kelly's facilitation of MP3 consisted of both whole class and independent work times involving partner conversations to provide her students opportunities for sharing, listening and reading ideas of others. A significant amount of math instruction time was spent with the whole class in which Kelly also provided opportunities for students to compare and critique ideas and to defend their ideas. It is also during whole-class instruction Kelly attended to facilitating discussion and maintaining an environment for students to engage in MP3.

Opportunities for students to share, listen to or read the solutions of others. A major element of Kelly's facilitation of MP3 was spent providing opportunities for students to share and explain their ideas and listen to others' solutions. Kelly believed this played an important

role in student learning because it allowed students to see and encourages the use of multiple strategies for solving math problems. Kelly expressed how her students "can use different strategies to get them to understand the concept" (Interview, p. 1, line 14-15) by providing them opportunities for turn and talks with shoulder buddies, or go to the board to explain their solutions for whole class analysis and critique. In part, this required students to be positioned as creators of ideas and intentionally selecting students to present different strategies,

> I usually pick students who got the correct answer and use different strategies from others so I might call up a student who used the quick 10 strategy and I might call up a student that used the place value charts or another strategy, but I want to expose them constantly to other strategies they can use to solve the problem (Interview, p. 7, lines 31-37).

This quote provided evidence of Kelly's intentional selection of students to present varying strategies for solving problems.

See and encourage the use of multiple strategies. Further, Kelly indicated, "whenever they see their peers use different strategies and knowing that a certain method may work, they might go back to their seat and try that different strategy that they just saw a peer use" (Interview, p. 2, lines 16-22). For Kelly, she exposed her students to multiple strategies and encouraged students to see and use strategies through both whole class and partner work.

During partner work, Kelly had her students use shoulder buddies for discussing multiple strategies. When Kelly provided turn and talk opportunities, she had her students "bring their whiteboards down to the carpet and so they can do their work on the carpet with me. And then I let them go off and turn and talk with their shoulder buddies and talk to them about how they solved the problem" (Interview, p. 6-7, lines 34-46 and 1-5). After discussions with shoulder

buddies, Kelly brought the class back together and partners shared out the highlights of their conversations. Kelly encouraged students to listen to one another by having them, "reiterate what their partner said back to me and to the rest of the class and explain what their partner did. After that, I'll ask the class if they agree or disagree with how they solved the problem if they got the right answer (Interview, p. 6-7, lines 34-46 and 1-5).

Kelly encouraged her students to use the most efficient strategy for themselves to ensure that all students were successful in math. Kelly indicated, "I don't teach to where they all have to use the number bond strategy, the arrow way or quick ten drawing. It is more like we have given them the tools needed to be successful in math and so whatever their most effective and efficient for them is what we encourage them to use" (Interview, p. 2, lines 32-38).

Use of manipulatives, drawings, or representations to support ideas. Kelly indicated for students to better understand their work and the work of others she, "usually has them share or show their work like a model and what they did and then bring their whiteboards up to the front and put it on the board and then tell us how they got their answer" (Interview, p. 8, lines 15-19). For Kelly's first graders, models, visuals and representations helped them explain their ideas to others as she expounded in a reflection, "I learned that my students do well when they can visually see the numbers and problems being done by using manipulatives or drawing a model. I noticed when students didn't draw their models that is when they were more likely to get an answer wrong" (Peer Video Group Reflection).

Kelly instructed her students to "draw me your models for your answers using quick tens" (Final Clinical internship Observation Notes) to support their solutions. In addition, she urged students to use specific math models such as place value charts, tens frames, number bonds. For example, she asked her students to use place value charts to show how they came to their

answers, “How do I write 10 tens on our place value chart?, How do I write this on my place value chart?, Do I put more in the tens box or the ones box? (Final Clinical internship Observation Notes). There were times during lessons when she supported struggling learners in using models. She recalled, “about halfway through the lesson we added the number bond to help students make connections to relating counting on and missing parts” (Supervisor 2 Observation Reflection).

Compare, contrast, critique, reflect and revise solutions. During whole-class instruction, Kelly provided opportunities for her students to compare, critique, reflect and revise their solutions and the solutions of others. Whole class instruction allowed students to consider others’ solutions by agreeing or disagreeing with others, comparing and critiquing correct and incorrect solutions, and deciding if others’ solutions make sense. While these opportunities occurred for whole class analysis and often included teacher modeled solutions, partner discussions specifically attended to explaining and comparing solutions.

Agree or disagree with peers. One-way Kelly provided students with the opportunity to think about the solutions of others was to ask students whether they agreed or disagreed with another peers’ work. This occurred in whole-class instruction after a student verbally stated an answer or showed and/or explained their work upon the board. During one lesson, Kelly asked her students to, “Did Max get it right? Show me with your thumbs if you agree, disagree, or don’t know” (Supervisor Observation Notes). Further in the lesson, Kelly asked students to agree or disagree with another students’ work and probed students who disagreed by asking, “Who can tell me why they disagree?” (Supervisor Observation Notes).

Correct vs incorrect solutions (counterexamples). Another way Kelly provided opportunities for students to compare and consider the solutions of others was by “having them

see where they went wrong in the problem and how they can do it better or different way to solve it, if that's what is most efficient, effective strategy for them to use" (Interview, p. 1, lines 39-43). During whole-class instruction, "we work through problems together on the carpet and I pull students who might not understand so much whether we're doing up to the front to display their work and have their peers critique it so they can better learn from their mistakes" (Interview, p.1, lines 31-36).

Kelly intentionally provided students with counterexamples, or incorrect solutions, for students to critique. During whole-class instruction, students provided counterexamples and correct solutions for side-to-side comparison as she described,

> I might call up a student who solved the problem incorrectly and have another student that solved it the same way but got the correct answer that way they can compare their work and see, okay this is where I made my mistake and help other students see that it is okay to make mistakes (Interview, p. 7, lines 37-41).

Again, here I noticed Kelly used visuals to support her first-grade students in verbalizing their ideas of why a solution may be wrong by asking, "Why is that one wrong? Come show me why it's wrong on the board" (Final Clinical internship Observation Notes). Many times throughout a lesson, Kelly instructed her students to "show me how to do a number bond, or show me on your whiteboards" (Supervisor Observation 2 Notes) to "see" students work without the requirement of verbalizing their thoughts.

Teacher modeled solutions. Kelly used teacher-directed instruction as a way for her to model how to solve math problems. She expressed, "I think that for students to learn, they have to see, and they have a good visual of what is expected of them. And so, who better to learn from than their teacher who is trying to get them to get to this goal" (Interview, p. 12, lines 15-18).

Intentional correct teacher models. During whole-class instruction, Kelly had students compare their work with her correct modeled solution. Subsequently, she asked students, "Does yours look like mine? If yours looks like this put your thumbs up. If yours doesn't look like this what is different about it?" (Supervisor Observation Video Notes).

Intentional incorrect teacher models. Additionally, during whole-class instruction, Kelly intentionally provided her own incorrect solutions for students to critique. She also indicated how this serves as a formative assessment of student learning, "I will purposely make mistakes on the board to let me assess who sees my mistakes and how they can fix my mistakes. They think it's funny that the teachers made a mistake and then they can see what's wrong" (Interview, p. 12, lines 26-30).

Intentional incorrect student models. Kelly intentionally modeled a students' response incorrectly for students to clarify and probe them to think deeper about their answers. One lesson provided evidence of how Kelly facilitates this through the following discussion,

K(Kelly): What do you mean take fourteen? Take it and do what with it?

S (Student): Cross off.

K: Cross off like this? [crosses off the number 14 on the board]

S: No! [laughing].

K: Circle it?

S: No! Use a ten frame.

K: [draws a tens frame on the board] Like this?

S: Yes!

K: Fills up tens frame with a dot each frame and then add four dots outside the frame] Like this?

S: Yes, and cross off 8.

K: and cross off eight of them?

S: Yes!

K: Okay here's my tens frame and my four and we crossed off eight. What do we do next? (Supervisor 2 Video Notes)

Teacher models method not evidenced in student work. When Kelly wanted students to see a particular strategy for solving a problem that was not provided or displayed by students, she modeled the strategy for whole class analysis. She indicated after noticing students did not use a certain strategy, she "ended up doing it on the board to let them see there are other ways to do this" (Interview, p. 13, lines 1-3).

Establish and maintain classroom environment for facilitating MP3. Kelly established a safe environment for students to share and listen to math ideas while maintaining a mindset for learning from mistakes. A large part of Kelly's establishment and maintenance of a respectful classroom environment for sharing included knowing which students to engage in MP3 at specific times while ensuring that all students have a voice in math class.

***The mindset to learn from mistakes*.** Kelly fostered a classroom environment where students learn from their mistakes. She indicated, "my kids know that they learn best from their own mistakes" (Interview, p. 1, line 38) and stressed the importance of "allowing them to feel comfortable enough in the classroom to have a voice and to share out their answers. And then encouraging students to look at and hear what their peers are sharing out and help them improve or ask those questions" (Interview, p. 17, lines 19-23). Kelly expressed maintaining an environment relied on strategically choosing students to show their incorrect strategies which

"helped the other students see that there are mistakes and it's okay to make them and they are our friends who have done it the correct way that can help us" (Interview, p. 7, lines 41-44).

Knowing which students to be pushed and supported. For Kelly, fostering a safe classroom environment where students feel safe to share their ideas and in return, being open to having ideas critiqued hinged on knowing her student's math abilities and confidence with their math knowledge. She explained,

> It has to do with knowing your students cause I think I would say that I know my students fairly well and so I can put one of my higher students up there who didn't solve it right and I can ask them higher-level questions like how did you do that? Or why did you do it this way? And they can handle it, but if I put up a lower level ability student who may be struggling with a concept and I start asking these questions, they are going to panic and not do too well (Interview, p. 9, lines 1-9)

Kelly admits that not all her students engage in MP3 to the same level. Kelly takes student ability into consideration when facilitating MP3. She expected all students to be able to share their answers with peers whether in partners or for whole-class analysis. Student ability played a role in how each engaged in MP3. When asked if all her students engage in MP3 to the same degree Kelly responded,

> I wouldn't say that they all give it the same level because my higher students can critique each other's work and see the mistakes, but then I have my lower students who you can explain how somebody did something wrong several times and they are still going to struggle in understanding it (Interview, p. 9, lines 34-43)

Additionally, student's confidence played a role in whether she chose them to share their ideas with a partner or share their ideas for whole-class analysis. She described how she engaged students with low confidence in opportunities to share their ideas,

> I am having them turn and talk to a partner, I listen to what they are saying. I am not going to call them up to the board because if I do call them up to the board and have students critique their work it's going to fall apart. I know there are other students that I can call on to share their answers and they are going to be comfortable sharing their answers and allow other students to critique their work (Interview, p. 6, lines 6-17).

Additionally, Kelly provided insight into the influence of ability on student engagement in MP3. She indicated language and capacity to express math ideas verbally is linked to students with lower math ability and thus influences engagement in MP3. Further, she stated students have the desire to share the ideas but may lack the verbal and/or language skills to do so. For Kelly, she allowed all students opportunity to share their ideas regardless of ability, "because they want to be able to share, but when it comes down to using the math language or sharing it with the purpose to critique each other's work they are not doing it for that reason" (Interview, p. 10, lines 1-7).

Facilitating whole class discussion for MP3. A significant amount of Kelly's math instruction occurred in a whole-class setting, therefore facilitation of MP3 was primarily evidenced during this time. Kelly's instruction for math was substantially teacher-directed using teacher modeling as support for students, she described, "when we complete a problem, we usually do one problem together and then I walk around the room monitoring on how they are coming along in solving the problem independently and if they need assistance, I can work with them one-on-one at that point" (Interview, p.2-3, lines 44-46 and 1-2).

Pattern structure for whole-class discussion in facilitating MP3. Kelly's structure for the facilitation of MP3 in whole-class discussion suggested a pattern that I have labeled, Question-Teacher-Partners-Independent-Share-Compare/Critique (QTPISC). The structure is illustrated in table 9.

Question or problem (Q). Kelly initiated math class with a contextual math problem similar to the one shown in the vignette. This problem was often displayed on the smartboard and emanated from a teacher-created PowerPoint centered on the district curriculum.

Teacher model (T). As evidenced in the vignette, after Kelly posed a problem or question to students, it is completed together as a class while Kelly modeled how to complete it using visuals, representations, or manipulatives. She explained, "I definitely teach the first ones on my own where I'm leading it. It's very teacher-led" (Interview, p. 5, lines 7-9).

Procedural questions were asked during this time to aid in how to solve the problem such as, "If I add one more bead how many do I have now?" "What's my answer if I had 6 tens and 9 ones?" "How many tens do I have?" "Do I put one more in the tens box or the ones box?" (Supervisor Final Clinical internship Observation Notes). Kelly explained, "during the example problem, questions were directed towards the whole class and were answered by the whole group calling out" (Supervisor 2 Observation Reflection). In a reflection, Kelly described how she and her students completed problems together as a class, "We then wrote our number sentence to find our missing part. We filled in the number bond with the total and two missing parts" (Supervisor 2 Observation Reflection).

Guided practice (P). Following the teacher modeled example, repeated teacher-guided examples were completed. Each example gave students a gradual opportunity to solve on their own. Kelly described, "we did this for several more problems, each student doing it on their own

personal number path and with their own bear. After doing several together, students began to do it on their own (Supervisor 2 Observation Reflection). During guided practice, Kelly provided students with opportunities to either share their solutions with a partner or with the whole class. Sharing ideas with partners was intended to have students discuss and listen to others' ideas with the possibility of coming up with a common solution, "After they had a moment to think about why I instructed them to turn and talk to their shoulder buddy about why they think we are putting our bears on the number eight. I called on a student to share their thoughts" (Supervisor 2 Observation Reflection).

Sharing ideas with the whole class was directed by Kelly and consisted of procedural questions similar to the ones asked in teacher modeling which supported students in completing problems step-by-step. In response, Kelly re-voiced, or repeated, students' answers while writing or drawing on the board. As illustrated in my observation notes,

> Kelly, re-voices the students' answer, "Nine, okay draw nine beads". She draws nine circles on the board as the students mimic her on their personal whiteboards. Kelly pauses as she waits for the students to finish drawing their circles. She looks around at various students watching them draw their circles (Supervisor Observation Notes).

Independent practice (I). During independent practice, Kelly explained, "once they are exposed to the content, I allow them to start doing it more independently" (Interview, p. 5, lines 10-12). Kelly spent her time walking around and monitoring student work by listening to conversations and helping struggling students one-on-one. Kelly described independent time as when students are individually working at their seats and when "I'm walking around the class and monitoring how they're doing and how they're solving their problems and I can talk to students individually if I see them struggling" (Interview, p. 4, lines 38-44).

Table 9

Kelly's Pattern Structure for Discussion in Facilitating MP3 (QTPISC)

Segment	Description	Example Evidence
Question (Q)	Math class initiated with a contextual story math problem	There were nine beads on the floor. A student picks up some of the beads, but there was still some on the floor. Write a number bond and number sentence that would match this story.
Teacher Model (T)	Teacher modeled how to complete problem or answer question using visuals, representations, or manipulatives	During the example problem, questions were directed towards the whole class and were answered by the whole group calling out
Guided Practice (P)	Completed repeated practice with teacher in whole class. Opportunities to share math ideas with partner or with whole class	We did this for several more problems, each student doing it on their own personal number path and with their own bear. After doing several together, students began to do it on their own After they had a moment to think about why I instructed them to turn and talk to their shoulder buddy about why they think we are putting our bears on the number eight. I called on a student to share their thoughts
Independent Practice (I)	Students work independently on math problems	I had the students work on the problem on their own and then they shared with their shoulder partner how they solved the problem
Share (S)	Students share out solutions with partner or for whole-class analysis	After I gave a few minutes for each one to talk, I asked students to share how their partners solved the problem
Compare/Critique (C)	Class compares answers with solutions presented by teacher or peers	I would ask them to share how they solved the problem and then I would write it on the board for the rest of the class. This allowed the students to see other's work and to compare it to their own, as well as find if the selected student had done the work the right way

Share (S). Following independent practice, Kelly provided students with the opportunity to share their ideas with others. First sharing with a partner and then for whole class analysis, "After I gave a few minutes for each one to talk, I asked students to share how their partners solved the problem. (Supervisor 2 Observation Reflection).

Kelly began whole class discussion for facilitating MP3 by asking questions that support students in explaining procedural steps for solving problems and how they came to an answer such as, "Did anyone come up with a set we can use?" (Observation Video Notes). Questions that foster student reasoning or defending solutions to develop mathematical thinking occurred less frequently during this time, however, some were evidenced throughout observation lessons such as, "Why are we starting at eight?" (Supervisor Level 3 Observation Notes) "Is this one less? Why not?" (Supervisor Final Clinical internship Observation Notes).

It is also evidenced and suggested that Kelly asked questions regarding procedure or process to those that struggle while reasoning questions were asked to those grasping the math content more easily. Questions such as, "What does a take from 10 strategy look like? What do you do first?" (Supervisor 2 Observation Video Notes) were asked to support students in explaining how they came to their answers. Kelly suggested that higher-level students can be pushed to answer questions that foster reasoning, however, students that might struggle benefit from procedure questions to explain how they got their answers. She explained, "I can ask those hard questions like, how did you do that? Or why did you do it this way? And they can handle it" (Interview, p. 9, lines 5-9).

Compare and critique (C). In response to students sharing their solutions for whole class analysis, Kelly provided students the opportunity to compare and/or critique the work of their peers.

> I would ask them to share how they solved the problem and then would write it on the board for the rest of the class. This allowed the students to see other's work and to compare it to their own, as well as find if the selected student had done the work the right way (Peer Video Group Reflection)

Comparing and contrasting answers included having students see how their solutions were same and different, "Myra, what did you do? She has ten and two, is that what you did? Is it the same as Ally's? What's different about it"? (Math Observation Video Notes). Opportunities for critique included deciding if solutions made sense and representing what is right or wrong about their own or another's solution. This is illustrated when Kelly asked the class, "Anthony has 111. Do we disagree? Who can tell me why they disagree?" (Supervisor Observation Notes).

Research question 3: In what ways do TCs perceive supports and tensions within their activity system? Learning that happens in an activity system is dependent on several components and supports achieving certain goals in collaboration with others. In looking at the factors influencing facilitating MP3, some components of practice-based learning came into focus (figure 8). Viewing Kelly's facilitation of MP3 through an Activity System lens highlights components that support the planning and facilitation of MP3 including intentional observations, CT observation and coaching, clinical internship school community, and practice-based coursework. Tensions existed within the system and that were barriers to facilitation of MP3 also emerged. Student behavior and ability levels, teacher-directed curriculum, clinical internship time constraints proved to be challenges for Kelly's facilitation of MP3.

Supports. Observations of teaching, coaching, co-teaching, and clinical internship classroom were indicated as supports influencing Kelly's facilitation of MP3. Particularly, Kelly's relationship with her first-grade CT played a positive role for significant coaching and observation. There were also indicators the practice-based math methods courses including peer video groups and observation of teachers using Bostic, Matney and Sondergeld's (2017) Look-Fors Protocol were effective in supporting Kelly's facilitation of MP3.

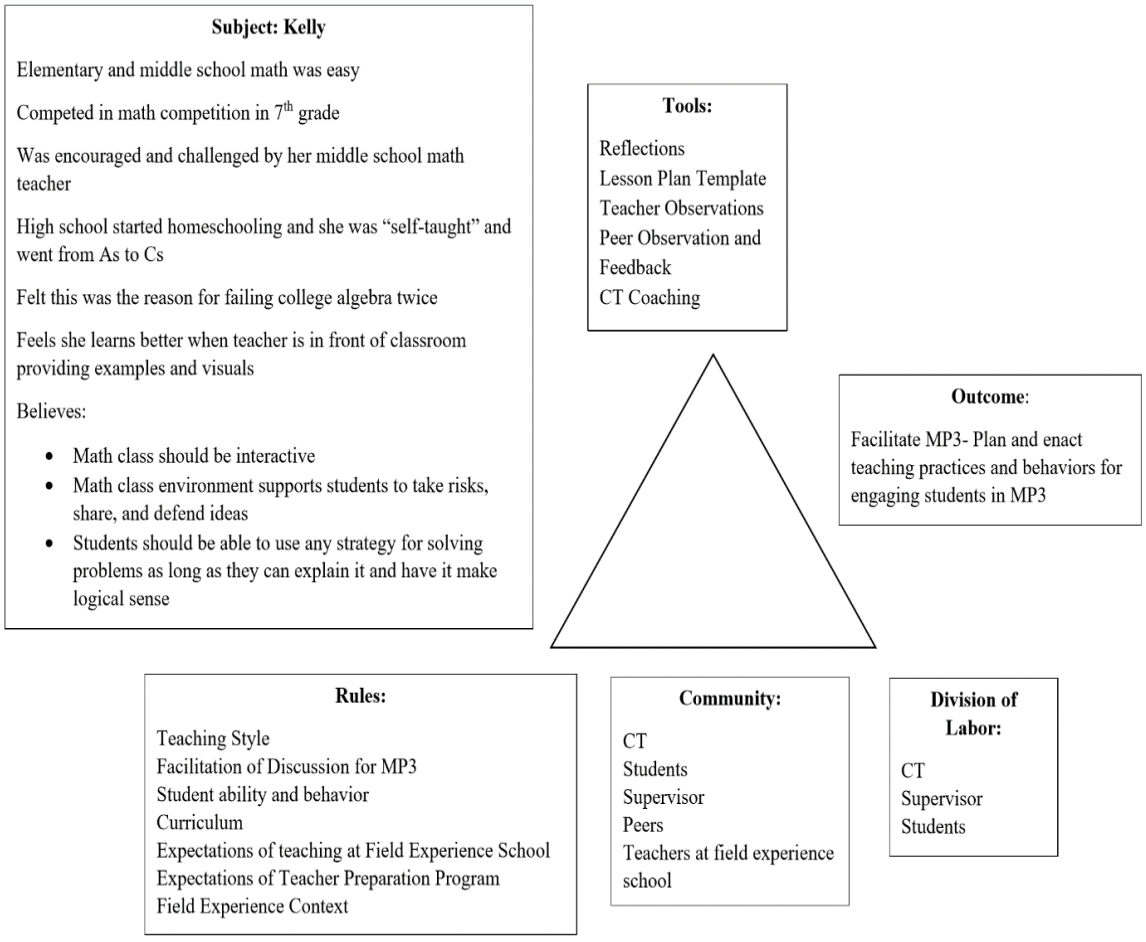

Figure 8. Kelly's Activity System for Facilitating MP3

***CT observation, co-teaching, and discussions*.** Kelly perceived her CT as a model for facilitating MP3. Observing her CT and noticing teacher behavior for facilitating MP3

influenced Kelly's practice, as she described, "watching my CT for the first several months of internship at the beginning of the year allowed me to see it modeled and she how she instructed her students and then how can I do the same thing in practice and learn what she's doing" (Interview, p. 13, lines 10-15). Additionally, the ability for sustained observation of math practice and the gradual acquisition of teaching math class played a role in Kelly's facilitation of MP3. She described,

> from August to December she taught math predominantly the whole time because I wasn't very comfortable teaching in whole group math instruction because last year, I used small groups almost exclusively. It was a time for me to really watch her and see how she doing it, how she was having the students engage in critiquing each other's work and turning and talking. I used those four months to really soak up what she was doing. And then when I came back in January, the first two weeks she still taught all the lessons, but then I began to gradually take over math. I used what I saw her do and using what I saw other teachers doing when I went in to observe them (Interview, p. 18, lines 7-22)

In early observations of her CT's math teaching and engaging in co-teaching methods, Kelly noticed and integrated intentional ways for facilitating MP3, "in the beginning we would take turns teaching math and she would constantly have students come to the board and I would be noticing okay, she is still doing this and I'm not so at that point I began to incorporate it into my lesson" (Interview, p. 13-14, lines 42-45 and 1-2).

Kelly specifically talked about a time where she and her CT discussed MP3, "we were talking about allowing time for turn and talks because, in the beginning, I wasn't allowing as much time as I am now. So, she did encourage me to allow time for those and to share out their answers" (Interview, p. 13, lines 36-38). Additionally, Kelly and her CT engaged in

conversations attending to student needs, “my CT gave me the lesson a few days before I taught it and we talked about expectations of the students and how to help support the students that would struggle the most. She allowed me to have access to the teacher's edition and any manipulatives I wanted to use” (Supervisor 2 Observation Reflection).

***Clinical internship school community*.** Kelly expressed the expectation of math teaching across her clinical internship school community,

> I think all teachers are held to a high standard of teaching here and asking those probing questions allowing students to talk about how they got their answers because that is what they are being evaluated on when they have walkthroughs done and so I think the whole school is trying to incorporate these practices in all math lessons, us teacher candidates too (Interview, p. 15, lines 27-32).

In part, the math teachers at her clinical internship school were evaluated and provided feedback by school leadership with an Instructional Practice Guide Walk-Through Tool aligned with district curriculum for teaching mathematics. Kelly had the opportunity to use the tool to visit other classrooms with her CT, she described the impact, “they also used the walk-through tool allowing me to see what they are looking for on that tool and how it looks in every grade. It helped me to see what teachers and students are doing in each part of it” (Interview, p. 13, lines 16-18).

***Practice-based math methods course*.** Components of the practice-based math methods course influenced Kelly’s facilitation of MP3 including intentional observations of math teachers with Bostic, Matney, and Sondergeld (2017) Look-Fors Protocol and peer video groups.

Intentional observations. Using intentional observations and reflections played an important role in Kelly’s facilitation of MP3. Kelly noted what she took away from one field observation

experience as she reflected on her knowledge of facilitating MP3 and what she was seeing in field observation classrooms with Bostic, Matney, and Sondergeld (2017) Protocol,

> For our math class, we came in and observed teachers teaching and writing down notes and specifically looked at what and when teachers were saying and how students responded. It helped me to see how important MP3 is in the classroom and how it can be used across different grades (Interview, p. 13, lines 21-28).

Additionally, Kelly indicated intentional observations allowed her to see MP3 facilitated in more than one way so she had a better understanding of MP3, "allowing us to see different teachers facilitating MP3 in different ways because every teacher does what's best for their class and what works best for them. So, going into different classrooms and seeing it used in so many different ways at different times was really helpful" (Interview, p. 17-18, lines 39-41 and 1-2).

Peer video groups. Kelly expressed her appreciation for engaging in peer video groups during the practice-based math methods course II on more than one occasion. During our interview during her final clinical internship, she stated, "When we video recorded our lessons and then provided feedback to our peers was helpful too. To have their feedback and then go back and watch their videos and give them feedback on how they did well or could improve on facilitating MP3" (Interview, p. 14, lines 32-38). Additionally, she admitted she was hesitant to engage in peer video groups at first, but subsequently valued the peer feedback she received,

> Honestly, I liked engaging in Peer Video Group more than I originally thought I would. In the beginning, when we were first assigned these groups I was hesitant because of it being awkward and uncomfortable for a peer to watch my teaching. But I actually really enjoyed their critique and input. I really appreciate the feedback that I received from both

> my peers and have considered the feedback that they gave me (Peer Video Group Reflection, p. 2).

This quote provided further evidence of the influence teaching representations and feedback, specifically from peers in her cohort and math methods courses, had on Kelly's facilitation of MP3.

Tensions. Tensions existed within the system and that were barriers to facilitation of MP3 also emerged. Student behavior and ability levels, teacher-directed curriculum, clinical internship time constraints proved to be challenges for Kelly's facilitation of MP3.

Student behavior and ability levels. Kelly struggled with attending to student ability level when teaching math and the need for differentiation. She realized for MP3 it's important that students see others work for learning however, she struggled with addressing the content for all levels of students in her class. It is important to note Kelly spent level 1 and 2 clinical internships in a co-taught classroom with two teachers and 30 students. It was structured with small group work and very little whole-class instruction. It was evidenced she may fall back on what was modeled for her in her clinical internships during this time. She expressed,

> in first grade my kids are quite a handful at times and I would say that sometimes from my lower students if I am trying to teach a more complex concept where they need to decompose two numbers like decomposing one number for a number bond my higher kids can understand, but my lower students were struggling and tended to be off-task and distracted. So, I think the students themselves make it a challenge because it's not on their level when I am teaching a whole group lesson. I would rather pull a small group for my higher kids in that area but I also want to expose the other students to it to understand that there are other ways to solve the problem (Interview, p. 3, lines 2-15)

As a result, one-way Kelly supported her "lower ability students and two that are special needs and are on the autism spectrum" was by paring students and using partner activities, "I usually either put them with a higher ability student or pair them up with me or my CT to make sure that they are getting extra support and trying to use the language or at least attempting it" (Interview, p. 5, lines 22-38).

Additionally, Kelly admits asking students to justify their answers or defend their thinking caused tension "because we are in first grade and it's challenging to get them to that point of questioning as to why they solved their problems that way or why they made decisions. We tried to get to that point, but I would say that it hasn't successfully gotten there yet" (Interview, p.7, lines 34-38).

Teacher-directed curriculum. Kelly explained how her grade level used a PowerPoint from a website that sells materials for math instruction, "we use a PowerPoint and we got it off Teachers Pay Teachers for the Curriculum. So, it's already bundled together". Kelly also indicated the "PowerPoint has some higher strategies and some lower strategies. And so using the PowerPoint and going through it also when they are doing independent work I am walking around the class and monitoring how they are doing and how they are solving their problems I can refer back to the PowerPoint to help struggling learners and I can say "okay you are doing this, how can we change this? Or what are we doing wrong here?" (Interview, p. 4, lines 27-44)

Clinical internship contexts. Kelly expressed time spent in her level 3 clinical internship classroom (2 days per week) was inadequate for allowing her to see how lessons build upon one another. She indicated interruptions in attendance in her classroom created tension in learning how to support learners by making connections to content either previously addressed or establishing knowledge built upon in future lessons. She reflected,

> One thing that I learned by doing this lesson was to help students make connections to prior knowledge or scaffold onto past lessons or what is coming ahead in future lessons. It is hard as a teacher candidate that only comes in a couple of days a week to know how to connect lessons because we aren't in there every day to know what has already been taught or what will be taught in future lessons (Supervisor 2 Observation Reflection)

Kelly's reflection provided evidence supporting the profound influence of being in clinical internship classrooms has on TC's teaching practice and learning to teach.

Case 3 summary. Kelly's initial lesson plans briefly indicated how she would engage students in MP3 with general behaviors attending to facilitation. Later lesson planning indicated further specific behavior for engaging students in MP3 more frequently. By final clinical internship, Kelly did not explicitly attend to MP3 in her written lesson planning, and there was no change in her level of implantation of MP3, suggesting facilitation may have needed less conscious planning as it became more automatic.

Kelly's facilitation of MP3 consisted of both whole class and independent work times involving partner conversations providing her students opportunities for sharing, listening and reading ideas of others. A significant amount of math instruction time was spent with the whole class in which Kelly also provided opportunities for students to compare and critique ideas and to defend their ideas. It was also during whole-class instruction Kelly attended to facilitating discussion and maintaining an environment for students to engage in MP3.

A major element of Kelly's facilitation of MP3 was spent providing opportunities for students to share and explain their ideas and listen to others' solutions. Kelly believes this played an important role in student learning because it allowed students to see and encouraged the use of multiple strategies for solving math problems. Kelly expressed how her students "can use

different strategies to get them to understand the concept" (Interview, p. 1, line 14-15) by providing them opportunities for turn and talks with shoulder buddies, or to come to the board to explain their solutions for whole class analysis and critique. In part, this requires students to be positioned as creators of ideas and intentionally selecting students to present different strategies.

A significant amount of Kelly's math instruction occurred in a whole-class setting, therefore facilitation of MP3 was primarily evidenced during this time. Kelly's instruction for math was substantially teacher-directed using teacher modeling as support for students. Kelly's structure for the facilitation of MP3 in whole-class discussion suggested a pattern that I have labeled, Question-Teacher-Partners-Independent-Share-Compare/Critique (QTPISC).

Observations of teaching, coaching, co-teaching, and clinical internship classroom were indicated as supports influencing Kelly's facilitation of MP3. Particularly, Kelly's relationship with her first-grade CT played a positive role for significant coaching and observation. There were also indicators the practice-based math methods courses including peer video groups and observation of teachers using Bostic, Matney and Sondergeld's (2017) Look-Fors Protocol were effective in supporting Kelly's facilitation of MP3.

Tensions emerged for Kelly addressing facilitation of MP3 including, student behavior and ability levels, teacher-directed curriculum, and level 3 clinical internship context**.** Kelly struggled with attending to student ability level when teaching math and the need for differentiation. She realized for MP3 it's important that students see others work for learning however, she struggled with addressing the content for all levels of students in her class. Kelly expressed time spent in her level 3 clinical internship classroom (2 days per week) was inadequate for allowing her to see how lessons build upon one another.

Chapter 5: Cross Case Analysis, Implications, and Conclusion

Cross Case Analysis

Through this multiple case study intended to describe ways TC's facilitated MP3 across a practice-based methods course and clinical internship classrooms. Further, this descriptive multiple case study allowed me to "find out firsthand what each individual case does" (Stake, 2006, p. 27) and then, how TCs' experiences facilitating MP3 compared to each other. I used the components of activity theory (object, subject, tools, community, division of labor, and rules), as the lens through which to analyze TC's facilitation from the perspective of those involved by "appreciating the uniqueness and complexity of the case, its embeddedness, and interaction with its contents" (Stake, 1995, p. 16). Additionally, the investigation into TCs' experiences planning and enacting mathematical teaching behaviors for facilitating MP3 revealed challenges and struggles that are important for teacher educators to understand. This study was guided by the following questions:

- How does the TC plan to facilitate MP3 in their clinical internship classrooms?
- How does the TC enact teaching behaviors for facilitating MP3 in their clinical internship classrooms?
- In what ways do TCs perceive supports and tensions within the activity system of a practice-based methods course and clinical internship classrooms when facilitating MP3?

Chapter four provided an in-depth look at each case and I described themes of each TC's planning and enacting MP3 across math methods courses and clinical internships. Additionally, I considered the supports and tensions influencing each TC's activity system throughout the course of the study. I explored the ways TC's activity system components influenced their facilitation of MP3 over the course of Math Methods I and II courses and their corresponding levels of clinical internships. This chapter presents the cross-case analysis in which I first read through each case taking notes on findings, uniqueness of each case, and relevance of themes as they pertained to each TC's activity system. Then, I used constant comparative methods to look across the themes and patterns of each activity systems in which TCs facilitated MP3. The comparisons across each of TC's activity systems were guided by the following questions: What commonalities and differences are evident in the themes, relationships, and patterns across the three cases? How do these commonalities or differences explain the ways TC's facilitate MP3? What influences of the components of the activity system may explain these commonalities and differences? It is through this comparison that I discovered salient themes that cut across cases suggested in Figure 9 below.

Activity System

While different components of the activity system come into focus for each participant, I noticed salient themes emerge from the analysis pertaining to influences and interactivity within the system. In consideration of the activity system for facilitating MP3 across clinical internships and practice-based math methods course, I present the cross-case analysis through these components and describe their influence on TCs facilitation of MP3 (figure 9). For each component, I highlight themes and interactions influencing planning and enacting teaching behaviors for MP3. Further, I present supports and tensions influencing and interacting within

the system. First, I begin presenting themes across the TCs (subjects) and then consider the tools influencing their facilitation of MP3. Next, I present salient themes for division of labor or people sharing responsibility with TCs in the facilitation of MP3. Following, I describe the people within the community which play a role in influencing TC's planning and enacting teaching behaviors for MP3. Next, I present the rules or regulations that guide, limit and dictate the ways TCs facilitate MP3. I address the integral role students play within the activity system. Finally, I propose a trajectory for TC's facilitation of MP3.

Activity System as a Process for Facilitating MP3

As Anthony, Hunter, and Hunter (2015), Bailey and Taylor (2015), Ghousseini (2015), Campbell and Dunleavy (2016), and Ghousseini and Herbst (2016) found, it is very clear practiced based methods for facilitating teaching behaviors are beneficial for the TCs in this study. The cycle for learning how to facilitate MP3 used as the activity system was evidenced to be integral to TCs facilitation of MP3 (McDonald, Kazemi, & Kavanagh, 2013). Tools used in the Math Methods Courses to observe representations of math teaching, reflections of both observations and enactments of teaching behaviors for MP3 were influential for TCs in this study. Further, CTs played an important role both as a member of the activity system community and having shared responsibility for facilitating MP3. Additionally, the CTs' clinical internship classrooms were integral for providing TCs consistent and repeated opportunity to rehearse, receive feedback, and reflect on enactments of teaching behaviors for engaging students in MP3. It is important to note here, that although the activity system context initially intended to take place across both university and clinical internship classrooms, the data supporting TC's facilitation exclusively occurred in clinical internship classroom settings. As a result, I found the data in this study provided evidence of a process for planning and enacting teaching behaviors

for MP3 within clinical internships. I noticed the process, illustrated in Figure 10 below, emerged over the course of the study and throughout the TCs time in the teacher preparation program.

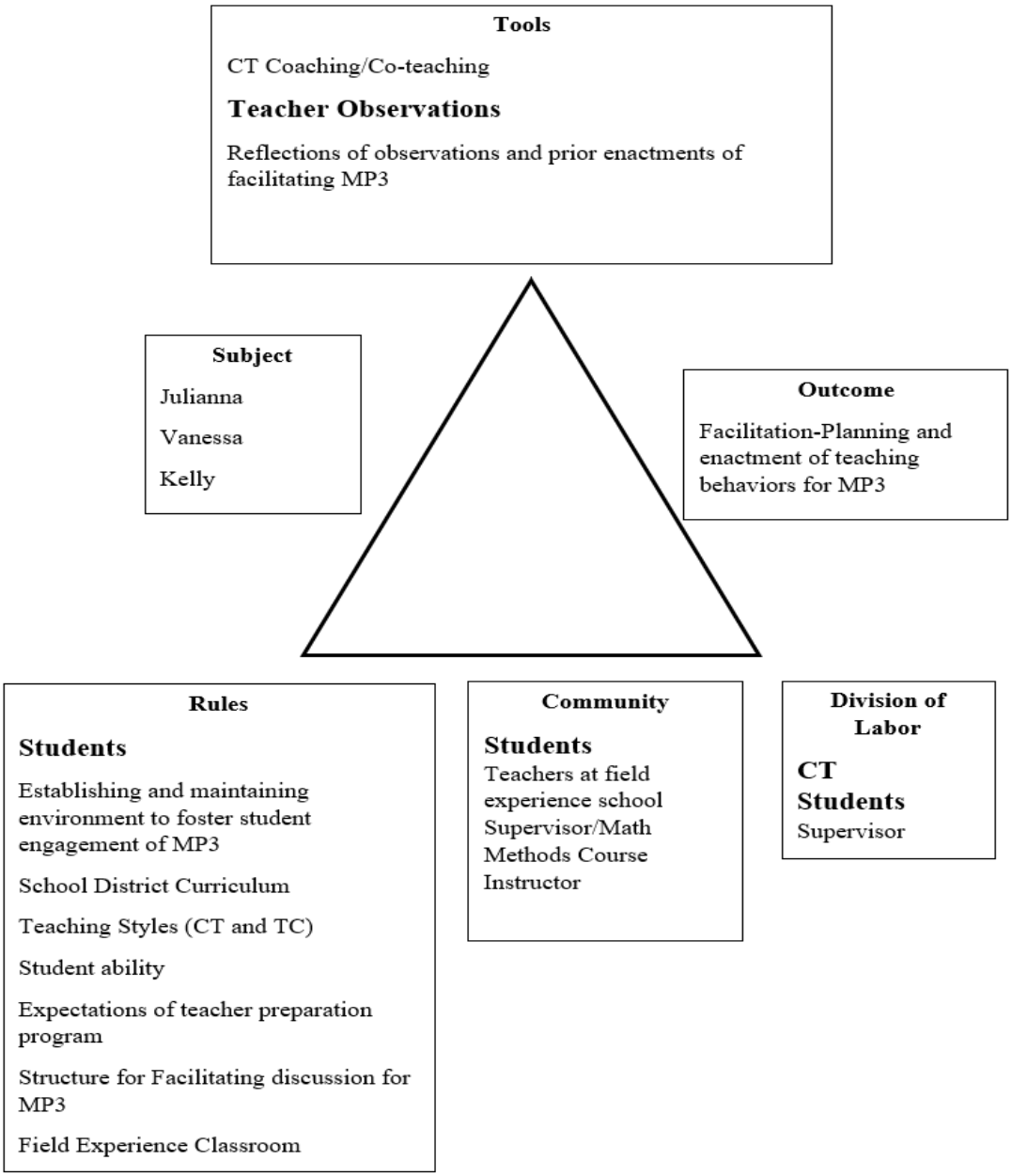

Figure 9. TCs' Collective Activity System for Facilitating MP3

While we know this process is not new, Lampert, et al. (2013) and Kazemi, et al (2016) used a similar cycle of enactment and investigation which allowed TCs to "travel back and forth

between methods course to enactment schools" (Lampert, et al, 2013, p. 228). I use it to describe the most influential experiences for the TCs in this study for learning to enact teaching practices for MP3 grounded in seeing and mimicking teaching practices. The process begins with observations of representations of teaching. Representations of teaching, as described by Grossman, Hammerness, and McDonald (2009), are intended to provide TCs with a visual or image of the teaching practices.

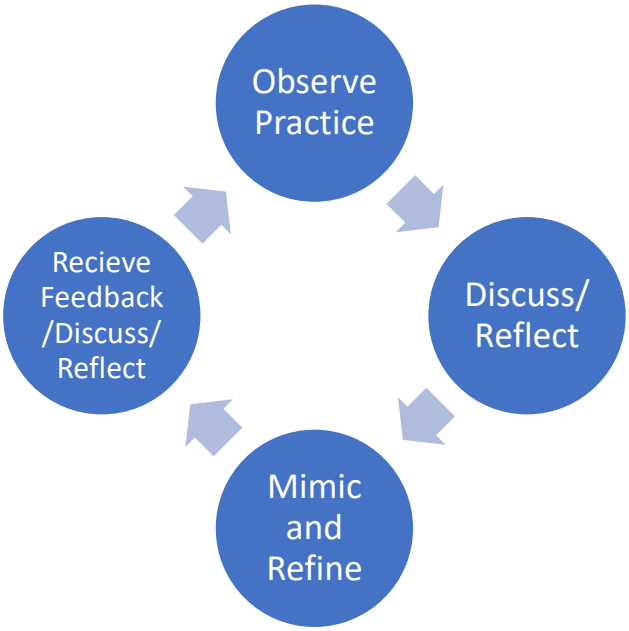

Figure 10. TC's Process for Facilitation of MP3

Following observations, discussions focusing on the representations to reveal any invisible actions and are important for digging deeper into how to engage in these practices. After these conversations, TCs are provided an opportunity to reflect on and make a plan for how they can apply the behaviors in their own practice. Now comes the time for TCs to rehearse or enact these practices within their clinical internship classrooms while being provided in-the-moment or immediate feedback. Again, TCs must then consider their enacting experiences and feedback to

think about any improvements to be made to their practice. Then the process begins again and repeats consistently over time until practices are refined and become a less conscious effort.

It is important to note, TCs repeated this process consistently throughout their two years in their clinical internship classrooms. By final clinical internship, this process was happening daily, and possibly, more frequently which provides supporting evidence for the importance of clinically-based teacher preparation programs. As the TCs provided evidence of the effectiveness of the process over time, it may be assumed that the process continues beyond teacher preparation programs and into their time as in-service teachers. This then may suggest an effective professional development model for improving teachers' practice throughout their time in the profession.

Subjects

Teaching is considered a cultural act and therefore, it is important to look across the TCs participating in this study as the subjects of the activity system (figure 11). Their prior experiences and beliefs about teaching play a role in how they each facilitate MP3. Comparing the TCs in this study, all three TCs had positive experiences in math throughout elementary school. Vanessa continuously has had positive math experiences throughout her secondary schooling and enjoyed mathematics both in school and at home. However, Julianna and Kelly had similar experiences struggling with mathematics in secondary school. All three TCs believe teaching elementary mathematics is grounded in students solving contextual math problems and encourage exposure and use of multiple strategies. Additionally, all three mentioned collaborative methods for teaching math allowing students to work together to solve and discuss problem solutions.

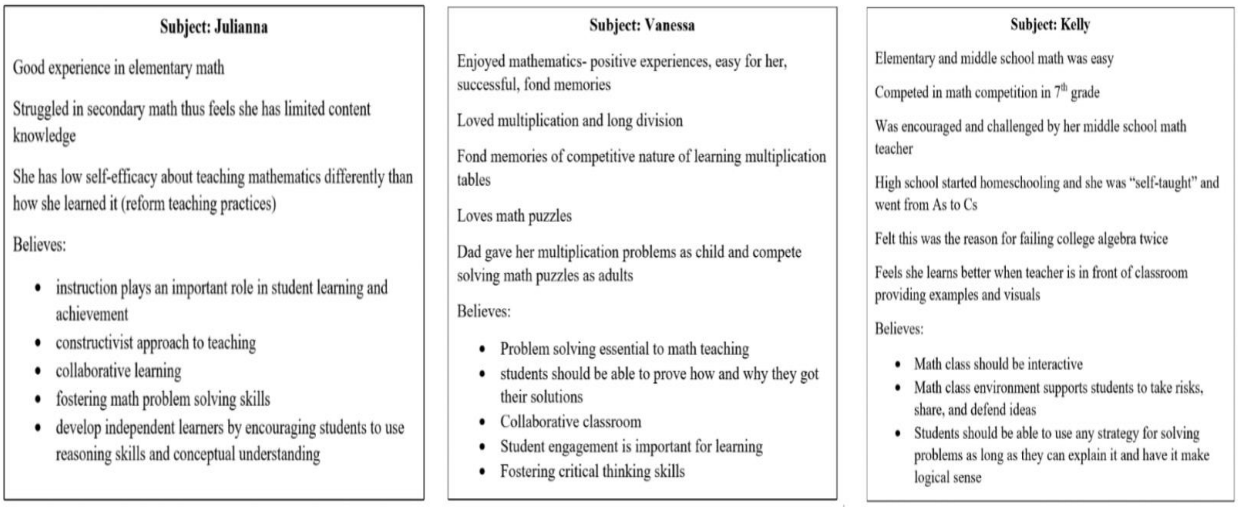

Figure 11. TCs as Subjects in the Activity

Tools

When planning and enacting teaching behaviors for the facilitation of MP3, TC's relied on tools of the activity system including reflections of prior enactments and rehearsals of facilitating MP3, and reflections of teacher observations of the facilitation of MP3. Additionally, the shared responsibility for lesson planning (division of labor) with their CT was indicated by all three participants.

Written lesson plan. In this study, I found the written lesson plan was not an effective tool for planning to facilitate MP3. In all three cases, TC's lesson plans provided general or brief statements showing knowledge of how to facilitate MP3. At some point, Julianna attended to more specific actions of responding to students and Kelly indicated a higher frequency of teacher actions for the facilitation of MP3. However, by final clinical internships, all three TCs provided limited information on lesson plans regarding teaching behaviors for MP3. There is evidence that may explain why the lesson plan was not used as a tool for planning, including teaching

practices' reliability on in-the-moment decision making, teaching behavior becoming habitual and less conscious, and influences from CT.

The written lesson plan was a requirement in the teacher preparation program for all levels of clinical internships and Math Methods Courses. The lesson plan does not specifically ask the TCs to address teaching behaviors for enacting MP3 and therefore may suggest that the written lesson plan, an expectation of the teacher preparation program, does not support the facilitation of MP3 in clinical internships. This may in-turn provide evidence of how the expectations of the teacher preparation program then become a regulatory piece dictating the way TCs plan for the facilitation of MP3, forcing it to become a rule in the activity system.

There seem to be additional reasons why the lesson plan tool was not influential on TC's planning to facilitate MP3 including, the reliability of in-the-moment decision making, practices becoming more habitual, CT influences on planning behavior. Johassen &Rohrer-Murphy (1999) stated "with practice and internalization, activities collapse into action and eventually into operations, as they become more automatic requiring less conscious effort" (p. 63). This may explain why by final clinical internship TC lesson plans became less effective for planning to engage students in MP3. Instead, TC's planning was dependent upon supervisor and CT coaching, shared planning work with CT, and reflection of earlier rehearsals and enactments of MP3.

Representations of teaching. As noted in the findings from Ghousseini (2015), Kazemi and Waege (2015), and Ghousseini and Herbst (2016), there is clear evidence that over the course of clinical internships TCs benefitted greatly from consistent intentional observation of teaching practices such as facilitating a discussion and then having the opportunity for consistent practice with explicit coaching (co-teach) and immediate, or in-the-moment, feedback on their

performance. Also, meaningful conversations surrounding planning and "How-to" addressing facilitation of MP3. TCs reported their CTs using question stems such as, "did you notice…" and, "watch me…" when engaging in these conversations.

Using intentional observations and reflections with Bostic, Matney Bostic, and Sondergeld (2017) Look-fors Tool plays an important part in having TC notice the many ways that facilitating MP3 plays out in classrooms. The tool chunks enacting MP3 into four broad teaching behavior indicators. Field observations provided TC with visual examples of what MP3 looks like in the classroom, provide ways for TC to emulate teaching, provide a repertoire of methods for facilitating MP3, understand the practices for facilitating MP3 how they look together in a math class, how to behave, how to respond to students.

It is important to note the Look-Fors Tool (Mattney, Bostic, & Sondergeld, 2017) described behaviors that expound behaviors in K-12 classrooms. From these three cases, it is suggested novice elementary teachers need a more scaffolded approach to learning these behaviors. Perhaps even a more detailed version that depicts teaching behaviors used with students across elementary grade levels. As it was suggested by both Vanessa and Kelly, facilitating MP3 mainly consisted of supporting students to verbally explain how they got their answers step-by-step. This suggests students, beginning in Kindergarten, need a scaffolded approach in learning how to verbalize their explanations then as they progress through the grade levels making connections how this lends to justifying and defending their solutions.

Reflections and discussion of representations of teaching played a role in "unpacking teaching in ways that give students access to the pedagogical reasoning, uncertainties and dilemmas of practice that are inherent in understanding teaching as being problematic" (Loughran, 2007, p. 6). This is important for making visible the teaching behaviors and their

influence on further teaching practices involved in engaging students in MP3. As documented by Bailey and Taylor (2015), reflections played an integral role to notice and compare teaching behaviors TCs planned to enact versus their actual enaction, particularly for Vanessa, named The Reflector. In-the-moment or pre/post-observation discussions with CTs, were more influential for both Vanessa and Kelly, to think about teaching behaviors for MP3.

Division of Labor

Division of labor, or those that share responsibility in TCs facilitation of MP3 focused on relations with CTs. The brunt of the work for planning and enacting behaviors of MP3 fell on the TC and their CTs. Although I, as the supervisor, met with TCs for pre-conferences of observation lessons and supported TCs in planning behaviors for engaging students in MP3, these conferences only happened a few times a semester. Further, I did not have immediate influence during the actual enactment of teaching behaviors during lessons.. CTs supported TCs more frequently in planning and enacting teaching behaviors for MP3 throughout their time in clinical internships.

CT shared responsibility for planning. The TCs were active in planning lessons collaboratively with their CTs. In talking with Kelly during her pre-conference she referred to her CT's ideas for the lesson which indicated a prior conversation took place about planning the lesson. She stated, "one idea she had was to have them put all their answers on the board using it as a model and have them explain how they got their answer and then having them figure out for themselves whether the answer is right or wrong" (Pre-conference Transcript, p. 3). Additionally, CTs played an important role in planning as models for planning, their teaching styles, interpretation of school district curriculum, and classroom management techniques influenced how facilitation of MP3 occurred. Another indicator evidencing the influence of CT for planning

was the TC's frequent use of the term "we" when talking about planning. The term "we" represented a collective effort between CT and TC.

There was no evidence to support CTs explicitly planning out teaching behaviors for MP3. Therefore, if CTs were not modeling actions of planning for engaging students in MP3 then TCs weren't exposed to it either. If we consider engaging students in MP3 as an unconscious effort (Johassen & Rohrer-Murphy,1999), then it may be harder for CTs to make this practice explicit to TC. So, it is up to TC to notice this practice in observations of CTs' teaching practice themselves. This requires TCs to be knowledgeable about MP3 teaching behaviors so they can intentionally look for the behaviors and actions facilitating MP3 requires.

CT shared responsibility for enacting behaviors for MP3. As illustrated in the process for learning to enact teaching behaviors, CTs were influential models for representations for teaching and behaviors for engaging students in MP3. They also engaged TCs in discussions and feedback around these behaviors. All three TCs had meaningful relationships with their CTs. They worked closely and had consistent significant conversations focused on teaching particularly those practices related to MP3. CTs were supportive in situations for risk-free practice and facilitation of MP3 which allowed for building confidence in the classroom. TCs and their CTs used co-teaching methods and CTs were available to jump in and support them if needed. This also allowed TCs to observe their CTs engage in practices related to the facilitation of MP3 while receiving in-the-moment feedback and coaching.

Models in representations of teaching. All three TCs perceived their CTs as models for behaviors of engaging students in MP3. TCs provided powerful quotes to provide evidence of the powerful impact their CTs had as representations of teaching. Kelly described,

> It was a time for me to really watch her and see how she doing it, how she was having the students engage in critiquing each other's work and turning and talking. I used those four months to really soak up what she was doing. And then when I came back in January, the first two weeks she still taught all the lessons, but then I began to gradually take over math. I used what I saw her do and using what I saw other teachers doing when I went in to observe them (Interview, p. 18, lines 7-22)

Additionally, Vanessa indicated that her CT afforded her opportunities to discuss and observe her teach math provided specific teaching behavior as an example for working with children,

> My CT let me watch her teach the prior lesson so that I could observe what kind of scaffolding that she used as well as how the students responded to the concept of imaginary fingers. I also discussed the lesson multiple times and she answered all of the questions that I had. In addition, she also asked me questions about how I was going to facilitate the lesson and different consideration ideas that she thought I should consider (Supervisor 2 Observation Reflection, p. 2).

Discussions with CT. Kelly specifically talked about a time where she and her CT discussed MP3, "we were talking about allowing time for turn and talks because in the beginning, I wasn't allowing as much time as I am now. So, she did encourage me to allow time for those and to share out their answers" (Interview, p. 13, lines 36-38). Additionally, Kelly and her CT engaged in conversations attending to student needs,

> my CT gave me the lesson a few days before I taught it and we talked about expectations of the students and how to help support the students that would struggle the most. She allowed me to have access to the teacher's edition and any manipulatives I wanted to us" (Supervisor 2 Observation Reflection).

Vanessa also specified using a co-teach model provided a scaffolded method for learning to teach alongside her CT,

> When I started teaching math, my CT would pick kids and have them go up to the board and kind of lead a discussion even though I was technically the one teaching math. But it was good to see how she led the conversation, which helped a lot. For me, it was the more I was in front of the kids the more confident I got. Once I felt like I knew what I was doing, it got easier to facilitate those discussions (Interview, p. 12, lines 3-11).

Observing their CTs teaching mathematics and having discussions around these observations provided all three TCs the ability to mimic and refine behaviors during her rehearsals of enaction in clinical internship classrooms.

Supervisor and math methods instructor as an influence. I, the Supervisor and Math Methods Course Instructor played a different role for each of the participants. For Julianna, her CT provided coaching of MP3 while Vanessa, I, as her Supervisor played a major support in planning for reform teaching methods including MP3 and important math content knowledge. Julianna had planning discussions with me as her supervisor to "ensure that I had a handle on teaching the material, how to address misconceptions, I am asking my students the right type of questions, and plan for if students aren't meeting the lesson objective" (Supervisor Observation 2 Lesson Reflection, p. 2, lines 1-9). Additionally, Julianna brought questions and concerns to be addressed in the conference to prepare for a successful math lesson and therefore "felt confident and prepared to go into her lesson" (Supervisor Observation 2 Lesson Reflection, p. 2, lines 13-16). It is important to note, this support provided by the supervisor occurred only two or three times throughout a semester.

Community

As illustrated in the sections above, it is clear that CTs were a large part of the community, or people who are part of each TCs' activity systems. All three participants had a close relationship with their CTs meaning that they got along well and felt comfortable in their classroom. Additionally, each communicated often and freely about teaching and personal matters with their CTs both in and out of school. For the most part, all CT demonstrated the facilitation of MP3 in their classrooms to some extent.

Rules

The rules of an activity system are those entities that guide, limit, dictate or regulate the ways TCs facilitate MP3. At the onset of the study, conventions of clinical internship classrooms were predicted to be the biggest impact guiding the ways TCs facilitate MP3. However, throughout this study, quite a few rules emerged influencing TCs planning and enacting teaching behaviors for MP3 including establishing and maintaining an environment to foster student engagement in MP3, school district curriculum, CT and TC teaching styles, and expectations of a teacher preparation program. This suggests the complex process of learning how to teach and the endless factors influencing novice teachers developing their practice. Looking across the three cases in this study, the most salient themes I noticed emerge from the data for the rules impacting TC facilitation of MP3 include teaching styles and school district curriculum. Here, the rules focusing on CTs seem to dictate what each of these looks like for TCs.

Teaching style. Both Julianna and her CT had a collaborative teaching style and was evidenced through the way they established and maintained a collaborative classroom environment for teaching math. Julianna provided evidence of facilitating MP3 in both whole class and small group discussions. She was the only case that revealed a consistent collaborative

instructional practice beyond partners to include small group work. Further, Julianna's use of questioning was aimed at both revealing student thinking and attending to supporting students gain a deeper understanding of the math content. This difference could possibly be influenced by the third-grade versus first-grade ability to command language and ability to engage in discussion for the facilitation of MP3.

Vanessa was struggling to find a balance between her beliefs about teaching math and how she was observing her CT facilitate MP3. She expressed, "I feel that my CT shuts students down too quickly and only allows one student to share their answer" (Reflection of Field Observation 2, p. 2, lines 1-7). Vanessa and Kelly's CT engaged in more teacher-directed teaching styles as therefore was evidenced in how they facilitated MP3. Vanessa indicated how modeling solutions for problems provides students an example of how to do the problem, "it is easier if I show them what I expect them to do, then they will be okay and know how to do it" (Pre-conference, p. 7).

These ideas provide evidence to suggest CTs teaching styles and conventions of clinical internship classrooms dictate the ways TCs facilitate MP3. Again, suggesting the powerful influence of CTs within the activity system.

Enacting school district curriculum. TC's planning and enaction were influenced by their CT's planning and enaction of the School District Curriculum. Each CT enacted and adapted the curriculum in different ways as was evidenced in Kelly's reliability on PowerPoint for math lessons, "we use a PowerPoint that is already bundled together" (Interview, p. 4, lines 27-28). She indicated the PowerPoints did not explicitly provide opportunities for engaging students in MP3, "it doesn't tell you what to do, it's more as you see fit kind of thing" (Interview, p. 5, lines 5-8). She further explained how they use the curriculum as a guide, not something they rely on

meticulously, “the curriculum itself has a dialogue, and I don’t use it verbatim, but it’s a good guiding tool” (Interview, p. 16, lines 9-13).

Julianna described how she and her CT chose which problems from the curriculum to use in math class, “we kind of skip over some of the problems because we don’t feel like they are valuable or relate to what we are doing in the lesson” (Interview, p. 7, lines 9-11).

Vanessa struggled with the district curriculum and relying on the reform teaching practices it used to teach first-grade math concepts. She was always looking for ways to adapt the curriculum so she understood the content. She described how she and her CT “normally look at the exit ticket, and then whatever types of questions are in the exit ticket we have them do those questions for independent work” (Pre-conference Transcript, p.4). Vanessa’s CT also provided in-the-moment coaching about making decisions during the lesson, “when I was teaching, my CT stopped me and said half of them get it, this is redundant for them, the other half need way more help. She had me break them into groups so that the kids that needed help sat with me on the carpet” (Pre-conference Transcript, p. 3). This provided Vanessa with experience in monitoring student understanding and making in-the-moment decision making that may not adhere to the district curriculum.

Students

I address the integral role students play within the activity system as their interaction influences all other components. Students can be listed as a rule as TCs must consider students' ability and behavior to dictate how they plan and enact teaching behaviors for MP3. Additionally, students can be placed as part of the community as they are the people directly impacted by and participate in teaching behaviors for MP3. Vanessa and Kelly provide evidence for their consideration in establishing and maintaining an environment for facilitating MP3.

Students also share responsibility, or division of labor, for facilitating MP3 when engaging in opportunities to share their work, listen to or read the work of others, and make comparisons between them. Without their participation in the activity system, TCs would not be able to achieve the outcome, or facilitate MP3. All activity within the system was aimed at students and, ultimately their learning of math concepts.

In-the-moment decision making for responding to students. Through data analysis, I found evidence to provide enacting teaching behaviors for engaging students in MP3 were largely dependent on student actions and behavior for in-the-moment decision making. This exemplifies Lampert, et al (2013), Kazemi, et al ((2016), and Ghousseini (2017) findings that rehearsals are beneficial for practicing and learning in-the-moment instructional decision making with eliciting student thinking. Julianna described lesson planning dependent on student behavior in solving math problems and reliant on in-the-moment decisions. She indicated, "I look at student responses, I look at how they're doing with the lesson, and then I come up with ways to make sure they are understanding and I kind of break it down throughout the lesson, but I don't intentionally plan for MP3 during my lesson planning" (Interview, p. 6, lines 6-9). Further, Vanessa indicated, "just depends on what kid it is and how I'm going to talk to that kid to get them to, you know, engage with their peers. I don't intentionally plan for which students" (Interview, p. 7, lines 30-33). All three TCs monitor students' behaviors for engagement in discussions related to explaining their work and listening to others explain their work.

Student math ability and language. Julianna, Vanessa, and Kelly supported struggling math students in explaining their solutions through the use of procedural questions. Julianna and Kelly acknowledged they encourage all students to participate in discussions for MP3 and how they provide language support to students through re-voicing techniques.

Questioning for student ability. Kazemi and Waege (2015) found TCs that pursued questioning practices encouraged students to create justifications for their mathematical ideas. While all three cases provided evidence regarding questions addressing procedure or process questions (step-by-step) supports students to explain how they got their answers and may be used for supporting struggling students, Julianna specifically attended to students' deeper understanding of math concepts and conceptual understanding in addition to procedural understanding for solving math problems. Julianna explained, "if I notice students are struggling, I'm, going to ask them more questions of breaking down the problem and how do we start this process" (Interview, p. 5 lines 26-28), while "having students explain their answers is a challenge sometimes, it's hard to get them thinking conceptually rather than procedurally so I say, I know this works, but how does this work?" (Interview, p.3, lines 21-26).

Vanessa scaffolds student responses by asking probing questions to support students in revealing a deeper understanding of how they got their answers. Vanessa also uses re-voicing techniques or gives a play-by-play account of what the student is doing on the board. Vanessa explains, "generally, after a student's initial answer, I would ask additional probing questions to clarify their thinking and to help me understand where the student's thought process is" (Supervisor 2 Observation Reflection, p. 3). During this time, she focuses on procedure questions such as, "What did you do first?" and repeats what the student said, "she drew 15 circles, then she put a ten frame around ten of them and then she crossed off eight" (Observation Video Notes). Further procedural questions to reveal student thinking include, "Where did you get the two from? So how many do you have altogether?" (Teaching Video Notes, p. 1). Vanessa used process questions specifically when supporting struggling students, "Let's start at the beginning. Where did you put this number or what is this and point to something specific? If I can have

them kind of explain their thinking and why they are doing what they are doing and if they have any idea what to do" (Interview, p. 8, lines 10-15)

Supporting language abilities. Julianna attended to the language barriers in her classroom by becoming a translator. She rephrased or re-voiced students' responses so all students could understand, she explained, "when students explain their answers using math language, not all my students understand. But that's when I modify it. I had to modify what they say for everybody to understand" (Interview, p. 13, lines 40-44 and p. 14, lines 1-3). She also modeled, reinforced, and encouraged all students to use math language. She encouraged all students to use math language. She expected it from those she knew were capable of using math language, "if I know a student can, then I expect it from them" (Interview, p. 13, line 13). Additionally, for students that may be struggling with verbalizing their ideas or using math language, she understands their difficultly of using math language in their discussions.

Additionally, Kelly provided insight into the influence of ability on student engagement in MP3. She indicated language and capacity to express math ideas verbally is linked to students with lower math ability and thus influences engagement in MP3. Further, she stated students have the desire to share the ideas but may lack the verbal and/or language skills to do so. For Kelly, she allowed all students opportunity to share their ideas regardless of ability, "because they want to be able to share, but when it comes down to using the math language or sharing it with the purpose to critique each other's work they are not doing it for that reason" (Interview, p. 10, lines 1-7).

Establish and maintain an environment. For both Vanessa and Kelly, establishing and maintaining an environment was a theme presented in the data. Coincidently, both Vanessa and Kelly were in first-grade classrooms. Through Vanessa's tensions with establishing an

environment for sharing, she developed an understanding that students need to feel comfortable and confident in order for them to share their ideas with others. For Kelly, fostering a safe classroom environment where students feel safe to share their ideas and in return, being open to having ideas critiqued hinges on knowing her student's math abilities and confidence with their math knowledge. Kelly fostered a classroom environment where students learn from their mistakes. She indicated, "my kids know that they learn best from their own mistakes" (Interview, p. 1, line 38) and stressed the importance of "allowing them to feel comfortable enough in the classroom to have a voice and to share out their answers.

Vanessa recognized the importance of building relationships had on creating and maintaining a trusting environment for discussion. Further, Vanessa demonstrated an understanding of her students' needs to feel comfortable making mistakes and taking risks for engagement in MP3. Vanessa expressed the need for all students to feel comfortable and confident when sharing their ideas with others, particularly for whole group analysis. She indicated the importance for students to feel successful in math class and supporting struggling learners by "leading and guiding them. I do not want to crush their confidence but they can still feel successful (Interview, p. 5, lines 15-24). She acknowledged knowing which students have the confidence to be placed in a sharing situation while having their work be critiqued is important for students to feel safe sharing in the classroom. She indicated, "I can question them even if it's about being wrong. Even if their answer is completely correct, I can ask, 'well why didn't you do it this way?' and they can defend their answers a little bit" (Interview, p. 6, lines 1-6). This quote provided evidence of how Vanessa sees the importance of students' roles in facilitating discussion around MP3. Knowing which students can share, explain, and be pushed influenced Vanessa supporting an environment where students felt safe to share their ideas.

A Trajectory for Engaging Students in MP3

Looking across the three cases focusing on students as their own component, I noticed a pattern in the different ways Julianna, Vanessa, and Kelly engaged their students in MP3. The pattern structures used by TC to facilitate discussion for MP3 in this study seem to suggest a discussion-based trajectory for what MP3 looks like across primary grade levels. The first-grade cases of Vanessa and Kelly seem to suggest that facilitation of MP3 pivots on supporting students to explain how they got their answers and listen to others. Kelly admits asking students to justify their answers or defend their thinking caused tension "because we are in first grade and it's challenging to get them to that point of questioning as to why they solved their problems that way or why they made decisions. We tried to get to that point, but I would say that it hasn't successfully gotten there yet" (Interview, p.7, lines 34-38).

Further, they supported student's development of language and conversation conventions for connecting their explanations of how they got their answers to defending ideas and justifying answers. Vanessa and Kelly did not necessarily ask students to defend or justify their thinking but rather used a teacher-directed method for supporting and scaffolding students to explain their solutions. Student explanations were grounded in using manipulatives and showing their representations or drawings while TCs re-voiced or supported student language for explaining their solutions. Sequential or step-by-step procedural questioning techniques encouraged students to tell how they came to their solutions.

However, Julianna's structure for discussion of MP3 in her third-grade clinical internship classroom focused on discussing different strategies and ask questions that probe students' thinking. Her questioning, although used process questions to support struggling learners, was grounded in asking students to justify their processes for solving math problems. Julianna

demonstrated the ways in which she attended to students to think deeper about the math and reason about their solutions by asking reflective questions. These types of questions go beyond explanation for how students got their answers and were aimed to have students begin thinking about justifications for their work. Julianna's data also provided a structure for facilitating small group and/or partner discussion for MP3. This seems to suggest that by third-grade students developed conventions of conversation to be able to hold extended, independent, student-centered discussions around explaining math solutions.

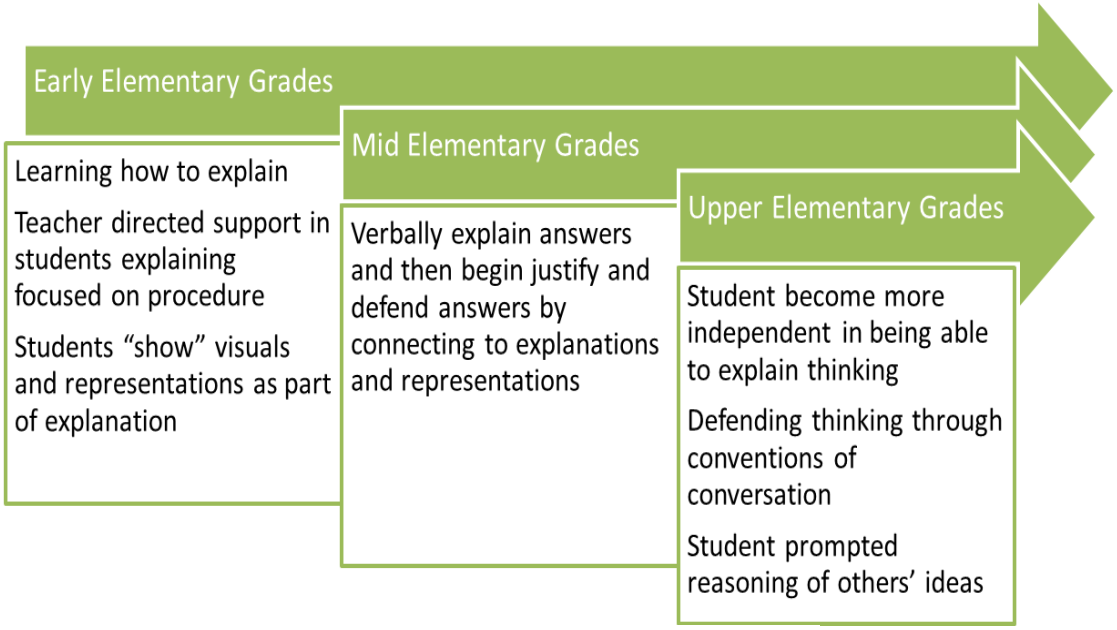

Figure 12. Trajectory of Engaging Students in MP3

Across all three TCs facilitation of MP3, they provided intentional opportunities for students to see multiple solutions whether they were student chosen strategies or teacher modeled strategies. This can include a comparison of correct strategies and also incorrect, or non-examples of solutions. They also provided opportunities for students to share, listen to, read solutions, see and encourage multiple strategies, compare correct and incorrect strategies,

exposed to nonexamples these indicators seem to be the foundation for facilitating MP3 in elementary grades.

This evidence seems to suggest that throughout elementary grade levels students are provided opportunities to see and encourage the use of multiple strategies for solving math problems. These opportunities allow students to share their solutions and ideas, listen to the solutions and ideas of others, and compare correct and incorrect solutions. Young students in primary grades learn to verbalize their actions through these experiences as they develop a command of math language and develop skills for conversations around math concepts. They begin to answer clarifying and probing questions based on their solutions. As students get older and begin to grasp these skills, then they can then start to verbalize their reasoning for decision making around their explanations while also critiquing the work of others. Students critique others' work by deciding whether the solutions are reasonable (are correct or incorrect), can tell why they are correct and incorrect, and be able to reflect on and revise any incorrect work. By the end of elementary school, students should be able to explain their solutions and provide mathematical evidence (argument) to support and justify why their solutions are correct/incorrect. Additionally, students should be able to engage in discussions in which they consider the work of others, answer and ask clarifying or probing questions for the purpose of critiquing, and decide if others' arguments make sense and are reasonable. These conversations should ultimately allow students to reflect on and revise their own work if needed.

A Trajectory of TC's Facilitation of MP3

This study provided evidence suggesting a conceptual model, or trajectory, for understanding the ways TCs (and novice teachers) learn to facilitate MP3 and possibly other Mathematical Practices. Figure 12 depicts a possible trajectory evidenced by the participants in this study.

Looking across the data, the participants in this study indicated they first began learning about and identifying broad behaviors encompassing MP3. As evidenced in their Math Methods I and II lesson plans, Julianna, Vanessa, and Kelly provided general or brief statements showing knowledge of facilitating MP3. For example, Julianna wrote, "ask students to explain solutions" (Planning Portfolio, p. 1 line 12). Also in the Math Methods II lesson plan, Vanessa indicated she would select students to share their sentences for whole class analysis by "discussing how you can make this a subtraction sentence, write it on your board and thumbs up when you're done. Call on students to discuss options. Have students write the correct subtraction sentence on their boards as I do, remind to circle the answer" (Peer Video Group Lesson Plan, p. 4). However, Vanessa did not identify any specific actions as to how she would facilitate these discussions or what options she wanted to address. Further in the lesson, Vanessa planned for sharing ideas by having "students participate by completing problems on their whiteboards and coming up to the board to demonstrate their work" (Peer Video Group Lesson Plan, p. 5). Again, Vanessa provided little to no details as to how coming to the board would play out during the lesson.

As TCs in this study learned more about the practices for facilitating MP3 and began to dig deeper in how to enact them (and heavily relied on what they experienced through interactions with CTs and their clinical internship classrooms), they provided further detailed action (sub-behaviors) for carrying out these practices. At this time, TC's may use methods such as, turn and talks or partner talk, thumbs up/thumbs down, or whole-class discussion which requires teachers to establish norms for discussing mathematics with others. Further, as each TC in this study learned more about facilitating discussion for the purpose of facilitating MP3, I noticed patterns emerge for the ways they were engaging students to share their ideas, which included questioning techniques, opportunities to share ideas with others, and listening to ideas of others.

As evidenced in Julianna's lesson plans during Math Methods II, to facilitate discussion for MP3, she indicated sub behaviors such as types of questions to ask, strategies for having students share ideas, talk with one another and compare answers, and approaches for students to see many ways for solving problems.

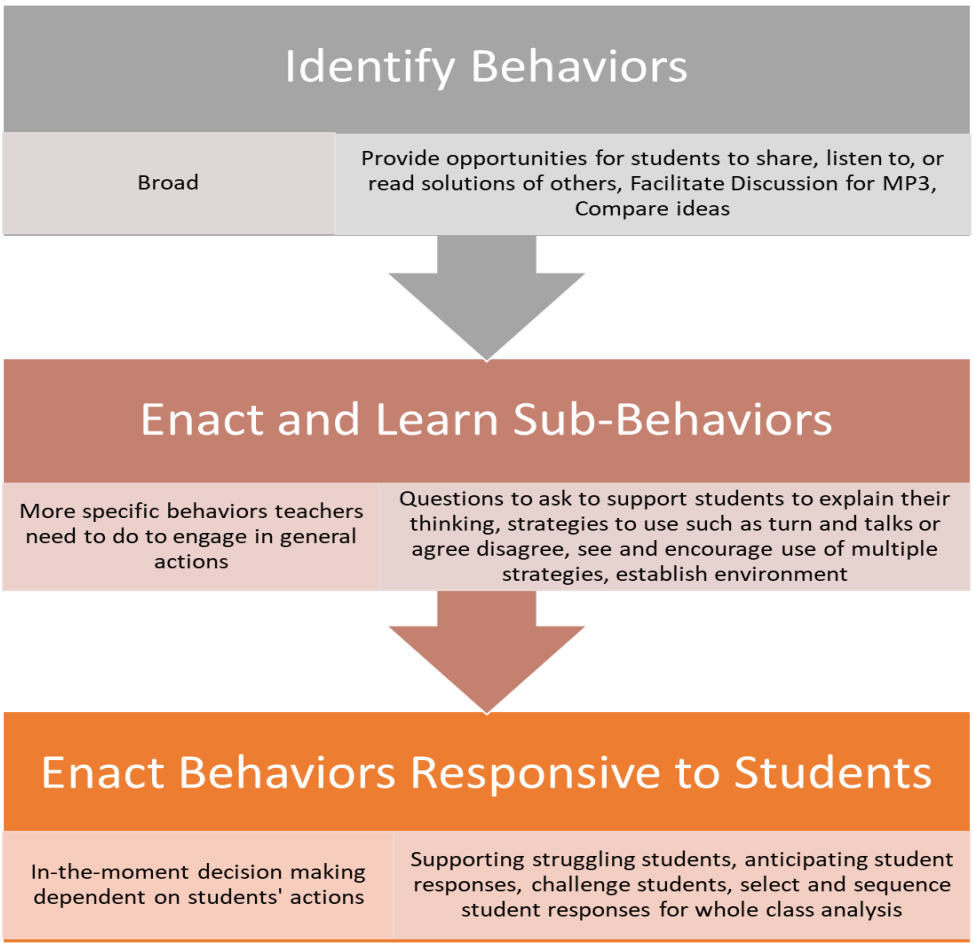

Figure 13. TC's Trajectory for Facilitating MP3

Additionally, Vanessa repeatedly supported and used teacher modeling to support the understanding of a variety of strategies in explaining the efficiency of methods with students, "When we were doing regrouping of 10, we talked about counting by tens, counting by 15, bundling to make ten, and other ways that they used. We talked about what was the most efficient method. We talk about the efficiency of them and I'll demonstrate and they demonstrate

and we figure out which one is the most efficient." (Interview, p. 9, lines 26-33). Additionally, Kelly expressed how her students "can use different strategies to get them to understand the concept" (Interview, p. 1, line 14-15) by providing them opportunities for turn and talks with shoulder buddies, or to come to the board to explain their solutions for whole class analysis and critique.

Finally, after the consistent, repeated practice of these behaviors over time, TCs were able to become responsive to students and make in-the-moment decisions during the facilitation of MP3. Additionally, with continual practice, TC's were able to internalize these actions thus becoming more automatic (Johassen &Rohrer-Murphy, 1999). Towards the end of their time in the program, TCs began to shift their facilitation from how to engage students in MP3 to how their facilitation applied and responded to their students. During the final clinical internship, Julianna demonstrated how her actions depended on students' behavior. She planned questions to support struggling students and how she would intentionally model specific solution strategies in the event students didn't demonstrate it themselves.

For example, during a lesson, Julianna encouraged students, specifically struggling students in engaging in MP3 by asking probing questions to help them to think about the step-by-step process in solving math problems.

Additionally, all three TCs described their intentional selection and sequencing of students to share for example, Kelly indicated "I usually pick students who got the correct answer and use different strategies" (Interview, p. 7, lines 31-32) while Julianna revealed, she intentionally chose students to address misconceptions and model a more effective way of solving problems. She also provided opportunities for students to see strategies with teacher modeling, "Then, as a class, I modeled to my students another way to solve a problem and explained why I chose that

way. This was whole class, therefore, each of my students were a part of this opportunity" (Supervisor Final Internship Observation 3 Reflection, p. 2, lines 34-37). Vanessa acknowledged knowing which students have the confidence to be placed in sharing situations while having their work be critiqued and "can question them even if it is about being wrong, even if the answer is completely correct, I can ask, why didn't you do it this way?" (Interview, p. 6, lines 1-6).

In table 10, I present more detailed behaviors depicted throughout the trajectory for TCs learning teaching behaviors to facilitate MP3. I have illustrated in this potential trajectory the general or broad teaching behaviors required to facilitate MP3 such as providing opportunities for students to share their ideas with others. Next, as TC's enact and interact with CTs and clinical internship classrooms they begin to break apart sub-behaviors needed to enact the broader behaviors. For example, in order to provide students with opportunities for students to share their ideas with others, teachers need to know how to facilitate discussion/ask questions and establish a classroom environment for sharing ideas and listening to others. At this time, TC's may use methods such as turn and talks or partner talk, or whole-class discussion which require teachers to establish norms for discussing mathematics with others. Finally, after the consistent, repeated practice of these behaviors over time, TCs were able to be responsive to students and make in-the-moment decisions during the facilitation of MP3.

Table 10
Trajectory of TCs' Behaviors for Facilitating MP3

Broad Behaviors for Facilitating MP3	Sub Behaviors for Facilitating MP3	Responsive to Students
Provide Opportunity for Students to share, listen to, read the solutions of others	Select and sequence students for sharing in whole class analysis Position students as authors/creators of math ideas and solutions where students show their work and explain how they got their answers Partner, small group, and whole class sharing situations (i.e. turn and talk) Knowing which strategies to highlight, encourage use of and model (teacher) Knowing which representations and encouraging use of visuals, manipulatives, and models to use	Knowing when and how to support struggling learners in sharing • Procedural Questions to support struggling students • Knowing not to give answer, but rather prompt students to think about procedural steps
Provide opportunities for students to see and encourage use of a variety of strategies and solutions	Intentionally select and sequence students for sharing for whole class analysis	Know which strategies you want students to see and intentionally provide opportunities for students to see them whether student modeled or teacher modeled Observe students and notice which solutions students are using (in-the-moment) Make decisions based on these observations for which student solutions to present to whole class and if any need to be modeled by teacher Ask higher level questions to make connections between these solutions and more efficient or abstract/advanced solutions

Table 10 (*Continued*)

Provide students opportunity to consider the ideas and solutions of others through comparing, critiquing, and revising solutions	Decide if others' arguments make sense and are reasonable compare: • correct solutions (multiples ways of solving) • correct and incorrect solutions (be able to say why a solution is correct and/or incorrect) • teacher modeled solutions (teacher intentionally provided solutions both correct and incorrect for student comparison and critique)	Encourage students to revise answers after considering others work (be able to amend/modify their own and others' incorrect solutions) Students become independent in doing this
Establish environment for facilitating MP3	students feel comfortable to learn from their mistakes, the mistakes of others, and to share their work Establish norms for students to critique one another	Knowing which students are confident to share and which ones can be pushed. Encourage students to ask questions about peers' work
Facilitate Discussion For MP3	small group and whole group discussion where students are sharing their ideas, explaining their ideas, determining reasoning of others ideas, comparing ideas types of questioning- what questions to ask Establish norms and classroom environment for collaboration and student conversations	Questioning- when to ask questions and to whom to ask questions to • procedural questions to support students in explaining how they got their answers and to support struggling students Patterns of questioning: questions to challenge students (higher level)- misconception? Why not all students get higher level questions? higher level questions to connect math concepts (did not see) student and teacher use of representations, visuals, and manipulatives
Provide students opportunities to engage in justification and proof of solutions and ideas	Prompt and provide tasks to encourage students to think critically about math concepts Ask higher level questions	

Summary

This multiple case study intended to describe ways TC's facilitated MP3 across a practice-based methods course and clinical internship classrooms. The comparisons across each of TC's activity systems were guided by the following questions: What commonalities and differences are evident in the themes, relationships, and patterns across the three cases? How do these commonalities or differences explain the ways TC's facilitate MP3? What influences of the components of the activity system may explain these commonalities and differences?

While different components of the activity system come into focus for each participant, I noticed salient themes emerge from the analysis pertaining to influences and interactivity within the system. It is very clear practiced based methods for facilitating teaching behaviors are beneficial for the TCs in this study. The cycle for learning how to facilitate MP3 used as the activity system was evidenced to be integral to TCs facilitation of MP3. Tools used in the Math Methods Courses to observe representations of math teaching, reflections of both observations and enactments of teaching behaviors for MP3 were influential for TCs in this study. Further, CTs played an important role both as a member of the activity system community and having shared responsibility for facilitating MP3. Additionally, the CTs' clinical internship classrooms were integral for providing TCs consistent and repeated opportunity to rehearse, receive feedback, and reflect on enactments of teaching behaviors for engaging students in MP3. As a result, data in this study provided evidence of a process for planning and enacting teaching behaviors for MP3 across both the Math Methods Courses and Clinical internships. While we know this process is not new, it describes the most influential experiences for the TCs in this study for learning to enact teaching practices for MP3 grounded in seeing and mimicking teaching practices.

When planning and enacting teaching behaviors for the facilitation of MP3, TC's relied on tools of the activity system including reflections of prior enactments and rehearsals of facilitating MP3, and reflections of teacher observations of the facilitation of MP3. Additionally, the shared responsibility for lesson planning (division of labor) with their CT was indicated by all three participants.

Division of labor, or those that share responsibility in TCs facilitation of MP3 focused on relations with CTs. The brunt of the work for planning and enacting behaviors of MP3 fell on the TC and their CTs. Although I, as the supervisor, met with TCs for pre-conferences of observation lessons and supported TCs in planning behaviors for engaging students in MP3, these conferences only happened a few times a semester. Further, I did not have immediate influence during the actual enactment of teaching behaviors during lessons. Therefore, I played a minor role while CTs played a larger role in facilitating MP3. CTs supported TCs more frequently in planning and enacting teaching behaviors for MP3 throughout their time in clinical internships.

It is clear that CTs were a large part of the community, or people who are part of each TCs' activity systems. All three participants had a close relationship with their CTs meaning that they got along well and felt comfortable in their classroom. Additionally, each communicated often and freely about teaching and personal matters with their CTs both in and out of school. For the most part, all CT demonstrated the facilitation of MP3 in their classrooms to some extent.

The rules of an activity system are those entities that guide, limit, dictate or regulate the ways TCs facilitate MP3. At the onset of the study, conventions of clinical internship classrooms were predicted to be the biggest impact guiding the ways TCs facilitate MP3. However, throughout this study, quite a few rules emerged influencing TCs planning and enacting teaching behaviors

for MP3 including establishing and maintaining an environment to foster student engagement in MP3, school district curriculum, CT and TC teaching styles, and expectations of the teacher preparation program. This suggests the complex process of learning how to teach and the endless factors influencing novice teachers developing their practice. Looking across the three cases in this study, the most salient themes I noticed emerge from the data for the rules impacting TC facilitation of MP3 include teaching styles and school district curriculum. Here, the rules focusing on CTs seem to dictate what each of these looks like for TCs.

I also addressed the integral role students play within the activity system. As an integral piece of the activity system. their interaction influences all other components. TCs facilitation of MP3 largely depended on students' math confidence and ability, conversation ability, behavior, and in-the-moment responses. Finally, I provide evidence to suggest a discussion-based trajectory for what MP3 looks like across primary grade levels. Beginning with younger children's facilitation of MP3 to support them to explain how they got their answers and listen to others' solutions. Later, teachers can attend to students to think deeper about the math and reason about their solutions by asking reflective questions. These types of questions go beyond explanation for how students got their answers and were aimed to have students begin thinking about justifications for their work. By the end of elementary school, students should be able to explain their solutions and provide mathematical evidence (argument) to support and justify why their solutions are correct/incorrect. Additionally, students should be able to engage in discussions in which they consider the work of others, answer and ask clarifying or probing questions for the purpose of critiquing, and decide if others' arguments make sense and are reasonable. These conversations should ultimately allow students to reflect on and revise their own work if needed.

Finally, I presented a possible trajectory for TCs learning teaching behaviors to facilitate MP3 which could also be applied across the other seven Mathematical Practices. This trajectory illustrated the need to learn about general or broad teaching behaviors required to facilitate MP3 such as providing opportunities for students to share their ideas with others. Next, as TC's enact and interact with CTs and clinical internship classrooms they begin to break apart sub-behaviors needed to enact the broader behaviors. For example, in order to provide students with opportunities for students to share their ideas with others, teachers need to know how to facilitate discussion/ask questions and establish a classroom environment for sharing ideas and listening to others. At this time, TC's may use methods such as turn and talks or partner talk, or whole-class discussion which require teachers to establish norms for discussing mathematics with others. Finally, after the consistent, repeated practice of these behaviors over time, TCs were able to be responsive to students and make in-the-moment decisions during the facilitation of MP3.

Implications

Teacher preparation programs. The findings I have presented in this dissertation have implications for teacher preparation programs and broadly address partnerships with K-12 schools and across content areas. First, in thinking about the importance of clinical internships and the influence of TCs' experiencing math teaching, establishing meaningful partnerships with P-12 schools is critical. Clinical work carried out in this study relied on valued mutual partnerships between the university and Mangrove Creek Elementary School. Administration, faculty and staff valued the work done for and with the TCs in this study. Partnership school administration should be open in allowing all teachers at theirs schools to become models for teaching. Teachers at partnership schools should to be considered teacher educators and trained to provide meaningful feedback (in-the-moment and immediate) and use co-teaching methods

for working with TCs. I also suggest ongoing research with co-teaching experiences between CTs and TCs to find methods for feedback and models that most benefit TCs' learning high leverage/core practices within clinical internships.

Further, this study highlights the collaborative nature in which university instructors, supervisors, and clinical internship school faculty should engage in when carrying out clinical experiences for TCs. I recommend course instructors become more collaborative not only with university supervisors, but also clinical internship school partnerships.

While this study addresses MP3exclusively, there are implications that cross core practices for other content areas. The idea that students justify and defend their ideas is not limited to math alone. Students are expected to support their answers in other content areas such as language arts and science. The methods for engaging students in discussions for supporting, justifying, and defending answers would be the similar in these content areas as well.

Additionally, the data collected across the activity system in this study provided evidence to support TCs' learning through the practiced based methods course and clinical internships. TCs' learning to facilitate MP3 and influence of CTs were grounded in the clinical internship settings and reinforces the necessity for teacher preparation programs to provide TCs sustained and consistent opportunities for learning about, rehearsal, meaningful feedback, and reflection of teaching practice. The powerful impact CTs have on TCs planning and enacting practices suggests the need for teacher preparation programs to carefully consider who TCs are placed with for clinical internships. Teacher preparation programs ought to be intentional about the schools and teachers chosen to work with TCs. They ought to ensure CTs are models for good teaching, make the act of teaching explicit, are able to provide meaningful feedback and engage in meaningful discussions with TC around the invisible pedagogies of teaching.

Teacher educators. The results of this study have some clear implications for teacher educators. The data suggest that learning practices for facilitating MP3 rely on the context of clinical internship classrooms to support in-the-moment decisions and being responsive to students. The trajectory for TC facilitation of MP3 presented in this chapter occurred over the course of this study which spanned the TCs' time throughout their program and suggests learning pedagogy needs to be prolonged over time, consistent, meaningful and happen with children in classrooms.

Supervisors across universities in the U.S. have a wide range of backgrounds, training, and experiences that may or not include mathematics education. Therefore, it is imperative that math teacher educators reimagine/reinvent their roles to not only embed their classrooms within PK-12 clinical internship classrooms but also become active in supporting TCs learning math content and pedagogy in those areas. For example, supporting TCs' lesson planning and providing feedback on TCs enactment and investigations of practice. As both the university supervisor and math methods instructor in this study, I was able to engage in a more active role in developing TC's facilitation of MP3 across the university classroom and clinical internship classrooms. More often than not, these roles are fulfilled by different people, therefore, we must think about how to better embed our math method coursework within fieldwork classrooms and/or work collaboratively to support TCs' learning and enacting teaching practices. Teacher preparation programs should consider who fulfills these roles and ensure and encourage collaboration to happen with supervisors and methods course instructors. Also, teacher preparation should expect and employ methods course instructors willing to reimagine their roles to span the boundaries of the program.

The roles of the university supervisor and math teacher educator crossed and blurred the boundaries of the university classroom and clinical internship schools. Additionally, evidence supports the powerful influence of CTs in the process for learning to facilitate MP3 described earlier in this chapter helps build TCs' knowledge of students while developing knowledge of content and curriculum. The process is dependent on clinical internships through which TCs intentionally observe teachers, ask questions and receive feedback about facilitation, notice teacher behavior, and practice/imitate teacher behavior. The evidence supports learning across these contexts overtime, teacher educators can begin to answer questions such as, how can we as teacher educators reimagine our roles and support this process in the university classroom?

There are implications from this study that would apply specifically to elementary math teacher educators. Math teacher educators should consider ways to construct and use of scaffolded observation protocols for intentional observations. These protocols should be focused on the trajectory of engaging elementary students in MP3 and highlight the developmental trajectory of teaching behaviors MP3. For example, initial math teacher observation protocols should support TCs in noticing broader teacher and student behaviors for facilitating MP3. Subsequently, protocols should progress in attending to more specific sub behaviors and finally notice behaviors for responding to students.

Teacher educator educators. There is limited research regarding the preparation and education of teacher educators and more specifically, mathematics teacher educators. The findings of this study provided evidence for the beneficial influence of clinical-based and practiced-based methods for working with TCs. This study could be used as an example of the ways teacher educators can span the contexts of university and clinical internship classrooms. Further, the ways methods courses can embed and support learning happened within clinical

internships. This study also highlights the knowledge and skills teacher educators need to engage in this work, including scaffolding TCs experiences in the field, and the noticing behaviors and core practices for teaching. This study prompts teacher educator educators to begin thinking about what knowledge is needed for this work and how we may go about learning it.

Collaborating teachers. Process for how TCs learn to enact teaching practices is grounded in experiences with CT. The CT perhaps play the most influential role in the process. They are the prominent person in the community of the activity system, their classrooms are essential components to the setting and context of the activity system, they are models in representations of teaching, their practices are mimicked by TCs, their feedback and coaching is integral for TCs professional development, and they share a large portion of the responsibility for TCs' facilitation of teaching behaviors.

CTs should consistently use co-teaching methods with TCs to provide in-the-moment coaching and feedback on teaching practices. As a result of the TC's finding conversations focused on facilitating MP3 with their CTs valuable, it is also recommended that CTs engage their TCs in conversations and discussions focusing on high leverage and core practices. These discussions should be based on the trajectories for engaging students in MP3 and facilitating MP3 across clinical internships. Conversations spanning clinical internships ought to scaffold behaviors for facilitating these practices and highlight teacher broad behaviors, sub behaviors, and response to student behaviors as they are being developed over time.

Inservice teachers. The trajectory for engaging students in MP3 is impactful on in-service teachers and those teaching younger students. This study began to highlight the ways students are supported to explain their answers and how this is linked to justification while attending to a deeper understanding of math concepts throughout the grade levels. As we begin to think about

linking our knowledge of children's language acquisition and development, the ability to have discussions, and how kids learn math we can better support students in engaging in MP3.

The process I noticed emerge from this study for facilitating MP3 can be a powerful professional development model for in-service teachers as well. The process of observing representations of teaching, discussion, and reflection are not limited to TCs or novice teachers. Math teachers can use this model if observations use tools grounded in learning standards, Common Core Standards for Mathematical Practices, and NCTM's Teaching Practices. Rich and meaningful feedback and discussions with colleagues around these tools following observations of teaching could provide important information for improving teaching practice.

Further Research

This dissertation provided a methodology for conducting an embedded case study where teacher educators can study have deep access from inside with a 360-degree view of learning across both university and clinical internship classrooms. The advantages of this method of case study provide the perspective of both the math methods instructor and university supervisor spanning across clinically rich contexts of the teacher preparation program. Further research using a similar methodology from the perspective of teacher educators embedded in the system can provide powerful information for the ways TCs learn to teach. Additionally, further research that might highlight the roles and influences of math methods course instructor and university supervisor within practice-based coursework for TCs facilitation of MP3 is needed.

Further study of the discourse between teachers and students would develop a better understanding of how teachers are supporting the deeper meaning of math concepts for students. Do elementary math teachers have enough math content knowledge to be attending to fostering students' deeper meaning of math content. TC provided evidence they know they had to ask

students “why they got their answers or why they made certain decisions for problem-solving”, however, what do know about how TCs make connections from students' answers to these questions and building a deeper understanding of math. And how this is done for all students. How do we support TC in developing both the content knowledge and pedagogical knowledge needed to do this? Transfer of skills?

Further research is needed in developing a trajectory for what MP3 looks like for students across elementary grade levels starting in Kindergarten (or younger). How these actions link together through the grade levels to get to secondary levels of proving events? And How facilitating discussion and conventions of language lend itself to this trajectory. How teachers scaffold MP3 across grade levels? And then how do we educate novice teachers to learn how to facilitate MP3 through the grades based on this trajectory? Additionally, I found that students play an integral part of the activity system in TCs facilitating MP3. There is liited research focusing on students role and influence in TCs’ learning core practices for teaching. Further research focusing on students’ role and influence could provide important information on how TCs learn to facilitate MP3 and other core practices.

Recordings of observations did not use microphones, and, as the camera was stationary throughout the TCs’ lessons, did not capture all teacher-student or student-student conversations. As a result, some discussions where students engaged in MP3 in explaining and comparing solutions were missing or inaudible. Perhaps if I had used multiple cameras and microphones, I would have captured more student-teacher discussions including questions and responses adding to the data supporting the ways TCs facilitate discussion and making in-the-moment instructional decisions dependent on student responses for MP3.

As stated in chapter 3, the intention of this research was to provide proof or evidence for TC's ability in engaging in teaching behaviors for facilitating MP3 as a critical first step in developing a model for understanding how novice teachers learn to facilitate MP3. This work considered the behaviors TCs have exhibited for facilitation of MP3 and does not look at what behaviors aren't exhibited. Further research is needed in considering what teaching behaviors and actions TCs are able and not able to do when facilitating MP3. This is a critical next step to further develop and understand a trajectory for learning to facilitate MP3 and possibly other Mathematical Practices.

The perspectives of TCs in this study were exclusively from female participants and no male perspectives were included in this research. Further research that includes male participants could provide further information regarding gender influences within the activity system. Julianna, Vanessa, and Kelly completed their clinical internship in the same elementary school and therefore, the data reflected the context of one clinical internship school setting. Further research of other clinical internship elementary schools could highlight and make visible the contextual influences of the activity system. Additionally, as noted above, language plays an important factor in facilitating MP3, for students to be able to explain and discuss math solutions. The percentage of ELL students at Mangrove Creek Elementary is small and not all TCs had ELL students in their clinical internship classrooms consistently over time in the program. Further research inclusive of experiences with ELL students and teaching mathematics, may provide data to support how language dynamics impact the facilitation of MP3 and as it pertains to that specific population of students.

Perspectives of CTs, who play an important role in the activity system for the facilitation of MP3, were not included in the data for this study. Further research including the perspectives of

CTs would highlight the factors influencing their role in the activity system. CT perspectives related to their decisions for enacting the curriculum and considerations the facilitation of MP3 could provide further information as to how they influence TCs facilitation of MP3. Further, the literature spanning practice-based methods is limited as to the role CTs play in supporting TCs learning high-leverage/core practices. Further research is suggested to explore CTs responsibility in practice-based methods of representation, approximation, and investigation of practice.

Conclusion

It is clear practice-based methods for learning and facilitating teaching behaviors for engaging students in MP3, constructing viable arguments and critiquing the work of others, are beneficial for the TCs in this study. The cycle highlighted for TC learning how to facilitate MP3 used as the activity system in this study was evidenced to be integral to TCs facilitation of MP3. Math Methods Courses providing TCs opportunity to observe representations of math teaching, reflect on both observations and enactments of teaching behaviors for MP3 proved to be influential. Further, CTs played an important role both as a member of the activity system community and having shared responsibility for facilitating MP3. Additionally, the CTs' clinical internship classrooms were integral for providing TCs consistent and repeated opportunity to rehearse, receive feedback, and reflect on enactments of teaching behaviors for engaging students in MP3.

Across all three TCs facilitation of MP3, they provided intentional opportunities for students to see multiple solutions whether they were student chosen strategies or teacher modeled strategies. This can include a comparison of correct strategies and also incorrect, or non-examples of solutions. They also provided opportunities for students to share, listen to, read

solutions, see and encourage multiple strategies, compare correct and incorrect strategies, exposed to nonexamples these indicators seem to be the foundation for facilitating MP3 in elementary grades.

The pattern structures used by TC to facilitate discussion for MP3 in this study seem to suggest a discussion-based trajectory for what MP3 looks like across primary grade levels. Further, they supported student's development of language and conversation conventions for connecting their explanations of how they got their answers to defending ideas and justifying answers. Student explanations were grounded in using manipulatives and showing their representations or drawings while TCs re-voiced or supported student language for explaining their solutions. Sequential or step-by-step procedural questioning techniques encouraged students to tell how they came to their solutions. This evidence suggested beginning in primary grades students learn to verbalize their actions. This also suggested that by third-grade students developed conventions of conversation to be able to hold extended, independent, student-centered discussions around explaining math solutions.

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Appendix A: IRB Approval Letter

USF
UNIVERSITY OF
SOUTH FLORIDA

RESEARCH INTEGRITY AND COMPLIANCE
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May 24, 2018

Andrea Willson
Teaching and Learning
Tampa, FL 33647

RE: **Expedited Approval for Initial Review**
IRB#: Pro00035301
Title: Exploring Preservice Teachers' Experiences Facilitating the Mathematical Practices

Study Approval Period: 5/24/2018 to 5/24/2019

Dear Ms. Willson:

On 5/24/2018, the Institutional Review Board (IRB) reviewed and **APPROVED** the above application and all documents contained within, including those outlined below.

Approved Item(s):
Protocol Document(s):
Protocol Version #1 4.23.18

Consent/Assent Document(s)*:
USF Student Consent Version 1 5.21.2018.docx.pdf

*Please use only the official IRB stamped informed consent/assent document(s) found under the "Attachments" tab. Please note, these consent/assent documents are valid until the consent document is amended and approved.

It was the determination of the IRB that your study qualified for expedited review which includes activities that (1) present no more than minimal risk to human subjects, and (2) involve only procedures listed in one or more of the categories outlined below. The IRB may review research through the expedited review procedure authorized by 45CFR46.110. The research proposed in this study is categorized under the following expedited review category:

(5) Research involving materials (data, documents, records, or specimens) that have been collected, or will be collected solely for nonresearch purposes (such as medical treatment or diagnosis).

(6) Collection of data from voice, video, digital, or image recordings made for research purposes.

(7) Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

As the principal investigator of this study, it is your responsibility to conduct this study in accordance with IRB policies and procedures and as approved by the IRB. Any changes to the approved research must be submitted to the IRB for review and approval via an amendment. Additionally, all unanticipated problems must be reported to the USF IRB within five (5) calendar days.

We appreciate your dedication to the ethical conduct of human subject research at the University of South Florida and your continued commitment to human research protections. If you have any questions regarding this matter, please call 813-974-5638.

Sincerely,

Kristen Salomon, Ph.D., Vice Chairperson
USF Institutional Review Board

Appendix B: Participant Consent Form

Informed Consent to Participate in Research Involving Minimal Risk and Authorization to Collect, Use and Share Your Health Information

Pro # _00035301____

You are being asked to take part in a research study. Research studies include only people who choose to take part. This document is called an informed consent form. Please read this information carefully and take your time making your decision. Ask the researcher or study staff to discuss this consent form with you, please ask him/her to explain any words or information you do not clearly understand. The nature of the study, risks, inconveniences, discomforts, and other important information about the study are listed below.

I am asking you to take part in a research study called:
Exploring Preservice Teachers' Experiences Learning and Facilitating Equitable Mathematical Practices

The person who is in charge of this research study is Andrea Scalzo Willson**.** This person is called the Principal Investigator. However, other research staff may be involved and can act on behalf of the person in charge. I am being guided in this research by Dr. Sarah Van ingen.

The research will be conducted at University of South Florida, College of Education.

Purpose of the study

The purpose of this study is to explore preservice teachers' experiences of learning to teach equitable mathematical practices.

Why are you being asked to take part?

I am asking you to take part in this research study because I would like to hear about your experiences in learning to teach mathematical practices. I would like to know what helped you in learning how to teach these practices and, if any, struggles that you experienced. I would also like to know what you think might have helped you better understand and learn these mathematical practices.

Study Procedures:

If you take part in this study, you will be asked to:

- Give consent for me to use math methods course assignments such as; written assignments, video observations, lesson plans and reflections as part of my research. Data

- will be striped of any identifying information and will be kept on a password protected computer only accessible to me. A pseudonym will be used to protect your identity, unless you specifically request that you be identified by your true name.
- Participate in one-on-one interviews where you will answer questions that will prompt you to describe your experiences in learning about the mathematical practices. The expected duration of the interview is 30 minutes. The interview will be completed at the University of South Florida's College of Education. I would like to video-record the interviews to make sure that I remember accurately all the information you provide.
- Vide-record observations of your teaching in the field at internship. Video-recording your teaching will make sure that I remember accurately all the information of the observation.

*Please know, I will be the only person to have access to these recordings and will keep them in a password protected computer which I will be the only one to have the passcode and they will only be used by me. The video recordings will be saved on the password protected computer for 5 years (IRB policy) after the Final Report is submitted to the IRB. When time comes, I will delete them from the computer's hard drive. If you prefer not to be video-recorded, I will use audio-recording instead. I may quote your remarks in presentations or articles resulting from this work. A pseudonym will be used to protect your identity, unless you specifically request that you be identified by your true name.

Total Number of Participants

Five individuals will take part in this study at USF.

Alternatives / Voluntary Participation / Withdrawal

You do not have to participate in this research study.

You should only take part in this study if you want to volunteer. You should not feel that there is any pressure to take part in the study. You are free to participate in this research or withdraw at any time. There will be no penalty or loss of benefits you are entitled to receive if you stop taking part in this study. Your decision to participate or not to participate will not affect your student status, course grade, recommendations, or access to future courses or training opportunities

Benefits

You will receive no benefit(s) by participating in this research study.

Risks or Discomfort

This research is considered to be minimal risk. That means that the risks associated with this study are the same as what you face every day. There are no known additional risks to those who take part in this study.

Compensation
You will receive no payment or other compensation for taking part in this study.

Costs
It will not cost you anything to take part in the study.

Conflict of Interest Statement
There are not conflicts of interest.

Privacy and Confidentiality
I will keep your study records private and confidential. Certain people may need to see your study records. Anyone who looks at your records must keep them confidential. These individuals include:

- The USF Institutional Review Board (IRB) and related staff who have oversight responsibilities for this study, including staff in USF Research Integrity and Compliance.

I may publish what I learn from this study. If I do, I will not include your name. I will not publish anything that would let people know who you are.

A federal law called Title IX protects your right to be free from sexual discrimination, including sexual harassment and sexual violence. USF's Title IX policy requires certain USF employees to report sexual harassment or sexual violence against any USF employee, student or group, but does not require researchers to report sexual harassment or sexual violence when they learn about it as part of conducting an IRB-approved study. If, as part of this study, you tell us about any sexual harassment or sexual violence that has happened to you, including rape or sexual assault, we are not required to report it to the University. If you have questions about Title IX or USF's Title IX policy, please call USF's Office of Diversity, Inclusion & Equal Opportunity at (813) 974-4373.

You can get the answers to your questions, concerns, or complaints
If you have any questions, concerns or complaints about this study, or experience an unanticipated problem, call Andrea Scalzo Willson, Principal Investigator at (315) 725-8293
If you have questions about your rights as a participant in this study, or have complaints, concerns or issues you want to discuss with someone outside the research, call the USF IRB at (813) 974-5638 or contact by email at RSCH-IRB@usf.edu.

Consent to Take Part in this Research Study And Authorization to Collect, Use and Share Your Health Information for Research

I freely give my consent to take part in this study. I understand that by signing this form I am agreeing to take part in research. I have received a copy of this form to take with me.

_____ _____

Signature of Person Taking Part in Study Date

_____

Printed Name of Person Taking Part in Study

Statement of Person Obtaining Informed Consent

I have carefully explained to the person taking part in the study what he or she can expect from their participation. I confirm that this research subject speaks the language that was used to explain this research and is receiving an informed consent form in their primary language. This research subject has provided legally effective informed consent.

_____

_____

Signature of Person obtaining Informed Consent Date

_____

Printed Name of Person Obtaining Informed Consent

Appendix C: Semi-Structured Interview Questions

1. To what extent do you believe that MP3 is important for student learning in your math class?
2. Can you describe what MP3 typically looks like in your math class? What are you as the teacher doing? What are the students doing?
3. Can you describe a math lesson where you facilitated MP3 successfully?
4. Can you tell me about a time where you found facilitating MP3 challenging? What was challenging about it?

Planning Questions

5. Do you plan to engage students in MP3 in all your math lessons? Why or why not?
6. To what extent do you think about MP3 when planning your math lessons? Can you describe how you specifically attend to MP3 when planning your math lessons?
7. To what extent do plan MP3 around student behavior, academic abilities, students' backgrounds, technology, instructional equipment, students with special needs?
8. Can you tell me about a time when you thought about these factors when planning to facilitate MP3?

Facilitating Questions

9. To what extent do you facilitate MP3 as you have planned it?
10. Tell me about some ways that you provide opportunities for students to share, listen, and discuss strategies or solution.

a. Do these opportunities also allow students to defend their ideas? Can you describe a lesson when students were given the opportunity to defend their ideas?

11. Tell me about the questions you ask which encourage students to defend their ideas and consider others' responses?
12. Tell me about how you encourage students to develop and refine their thinking or arguments?
13. How do you monitor student engagement in MP3? How do you know if students are engaged in MP3? What is the teacher doing? What are the students doing?
14. Do all students engage in MP3 to same level? Can you tell me about a lesson where all students were or were not engaged in MP3 to the same level?
15. To what extent would you be able to facilitate MP3 in a different grade level?

Activity Theory Questions

16. What are some limitations that prevent you from facilitating MP3 in your math class?
    a. Time constraints, student behavior, academic abilities, students' backgrounds, technology, other instructional equipment, students with special needs

17. Tell me about how you have learned to facilitate MP3. Can you tell me about some experiences that helped you learn how to facilitate MP3?
18. To what extent do student behavior, academic abilities, students' backgrounds, technology, instructional equipment, students with special needs influence the way you facilitate MP3?
19. To what extent has what you experienced in your internship classroom helped or hindered your learning to facilitate MP3? Can you tell me about a time when you had an experience in internship classroom when you learned to facilitate MP3?
20. To what extent has your CT played a role in learning to facilitate MP3?

a. How have your CTs teaching style, classroom community, beliefs informed your learning to facilitate MP3?
b. Does your CT intentionally facilitate MP3?
c. To what extent have you had conversation about MP3 with your CT?

21. To what extent have your peers influenced your learning of MP3? Can you tell me about a time when a peer influenced how you plan or facilitate MP3?
22. To what extent has the school community played a role in learning to facilitate MP3?

d. To what extent have you heard teachers in your Planning PLCs talk about facilitating MP3?

23. To what extent have other experiences played a role in learning to facilitate MP3?
24. To what extent did the Math Curriculum play a role in facilitating MP3? Questions to ask, how to facilitate MP3, how to facilitate discussion….
25. What other experiences do you wish you had to support you in learning and facilitating MP3?

Appendix D: Observation Lesson Plan Template

Lesson Content	
What Standards (national or state) relate to this lesson? (You should include ALL applicable standards. Rarely do teachers use just one: they'd never get through them all.)	
Essential Understanding (What is the big idea or essential question that you want students to come away with? In other words, what, aside from the standard and our objective, will students understand when they finish this lesson?)	
Objectives- <u>What</u> are you teaching? (Student-centered: What will students know and be able to do after this lesson)	
Cognitive Demand: How does my lesson enable students to closely explore and analyze math concepts(s), procedure(s), and reasoning strategies	
How does my lesson make student thinking/understanding visible and deep? How will you have students explain their thinking? How will you have students explain how they got their answers? How will students demonstrate their thinking?	
How does my lesson create opportunities to discuss mathematics in meaningful and rigorous ways (e.g. debate math ideas/solution strategies, use math terminology, develop explanations, communicate reasoning, and/or make generalizations)?	
Evaluation Plan- <u>How will you know</u> students have mastered your objectives? What are you listening for in student work and responses? What formative evidence will you use to document student learning during this lesson?	
What Content Knowledge is necessary for a teacher to teach this material?	

What background knowledge is necessary for a student to successfully meet these objectives? ☐ How will you ensure students' have this previous knowledge? ☐ Who are your learners? ☐ What do you know about them? ☐ What do you know about their readiness for this content?			
What misconceptions might students have about this content?			
Lesson Implementation			
Teaching Methods (What teaching method(s) will you use during this lesson?)			
Step-by-Step Plan (What exactly do you plan to do in teaching this lesson? Be thorough. Act as if you needed a substitute to carry out the lesson for you.) Where applicable, be sure to address the following: ☐ What Higher Order Thinking (H.O.T.) questions will you ask? ☐ How will materials be distributed? ☐ Who will work together in groups and how will you determine the grouping? ☐ How will students transition between activities? ☐ What will you as the teacher do? ☐ What will the students do? ☐ What student data will be collected during each phase? ☐ What are other adults in the room doing? How are they supporting students' learning? ☐ What model of co-teaching are you using?	Time	Who is responsible (Teacher or Students)?	Each content area may require a different step-by-step format. Use whichever plan is appropriate for the content taught in this lesson. For example, in science, you would detail the 5 Es here (Engage/Encountering the Idea; Exploring the Idea; Explanation/Organizing the Idea; Extend/Applying the Idea; Evaluation).

What will you do if…	**…a student struggles with the content?**
What will you do if…	**…a student masters the content quickly?**
Meeting your students' needs as people and as learners	**How does this lesson connect to the interests and cultural backgrounds of your students? How are you planning for all students to participate in the lesson? Do all students have opportunity to speak? Who holds authority for knowing mathematics? Do some students hold more status than others?**
	How will you differentiate instruction for students who need additional challenge during this lesson (enrichment)?
	How will you differentiate instruction for students who need additional language support? (i.e. use of language strategies- gesturing, use of objects, revoicing, graphic organizers, and manipulatives)
Accommodations (If needed) (What students need specific accommodation? List individual students (initials), and then explain the accommodation(s) you will implement for these unique learners.)	
Materials (What materials will you use? Why did you choose these materials? Include any resources you used. This can also include people!)	

Appendix E: Peer Video Group Lesson Reflection

Final Phase:
The end of the cycle of your Peer Video Group Lesson is reflecting on the process and how it has impacted teaching practice. This is a way to document your work and share your professional knowledge. THIS IS TO BE COMPLETED INDIVIDUALLY!!!

Directions:
The teacher candidate will present learning statements garnered from engaging in peer collaboration about the impact of the teaching mathematics. In addition, teacher candidates will draw conclusions about the process as a whole.

Reflection: Answer *ALL* the following questions and support your answers:

1. What did you notice best supported your student's development of concepts (e.g. teacher actions, use of tools, aspects of the task itself)?
2. What instructional strategies were effective? Ineffective?
3. What do you feel were the most successful aspects of this lesson? Least successful?
4. Did all students learn the mathematics you identify in your objectives? How do you know?
5. What was the impact on student learning by providing opportunities for students to listen and discuss mathematics?
6. What did you learn about your students' mathematical thinking?
7. Did all students have a chance to decide if their answers were right/wrong and explain why?
8. Did all students have a chance to decide of their peers' answers were reasonable?
9. How did engaging in Peer Video groups impact your teaching practice? What did you learn about teaching mathematics?

Appendix F: Simulation Lesson Plan Template

<table>
<tr><th colspan="4">Teaching Simulation Math Lesson</th></tr>
<tr><td colspan="4">Group Names:</td></tr>
<tr><td colspan="4">Grade Level:</td></tr>
<tr><td>What Standards (national or state) relate to this lesson?</td><td colspan="3"></td></tr>
<tr><td>Objectives/Mathematical Goals- <u>What</u> are you teaching? What do you want students to know and be able to do after this lesson?
Describe the exact learning expectations for students.</td><td colspan="3"></td></tr>
<tr><td>Assessment Plan- <u>How will you know</u> students have mastered your objectives? Describe tools, handouts, and techniques you will use. Don’t just write observation- describe what you will be looking for while observing students.</td><td colspan="3"></td></tr>
<tr><td>What Content Knowledge is necessary for a teacher to teach this material?</td><td colspan="3"></td></tr>
<tr><td>Focus on Mathematical Practice #3: Construct Viable Arguments and Critique the reasoning of others</td><td>Students:</td><td>Teachers</td><td>What questions will you ask to develop mathematical thinking:</td></tr>
<tr><td>Before: Describe how you will introduce the activity. Consider questions that will elicit students’ prior knowledge needed for this activity, get students curious about the task, and/or relate to their personal backgrounds or interests. What are the directions for getting started on the task?

During: Describe the expected actions of the students during this phase. What are they to be doing? How are you making sure each child is accountable? What will you ask students as you observe? (Ask good questions related to your objectives; don’t just say “good job!”) Describe possible extensions or challenges you will have ready for early finishers.

After: This is the most part of a lesson! What questions will you ask students that will help them understand the mathematics they explored in the task/activity? How will you structure those questions (e.g. think-pair-share, share with your elbow partner) so that all students will participate in answering each question? Will students be presenting findings? How will this be structured?</td><td colspan="3">Lesson Plan:</td></tr>
</table>

What misconceptions might students have about this content?	
What will you do if…	**…a student struggles with the content?**
What will you do if…	**…a student masters the content quickly?**
Materials	

Appendix G: Peer Feedback Notes

Your Name:	
Name of Person Whose Video is Being Reviewed:	
Peer Video Group Protocol Notes	
Please use this document for recording your notes for each peer's video. You will need a separate document for each video you watch. It may be easier to type directly into this document, so you can provide a copy to your peer and submit to the canvas assignment.	
Is this a task of lowest, low, high, or highest cognitive demand? How are students engaged in solving the task?	
How does the teacher provide opportunities for all students to discuss mathematics in meaningful and rigorous ways? How does the teacher provide opportunities for students to *listen* to the solution strategies of others, *discuss* alternative strategies or solution(s), and defend their ideas?	
How does the teacher provide opportunities for students to listen, read, and critique the arguments of others? How are students engaging with their peers' mathematical thinking?	
What questions were asked that encouraged encourage students to defend their ideas, consider student(s) response(s)?	
How does the teacher engage students *in proving events* that encourage students to explain "how they know" to develop and refine mathematical arguments?	
How does the teacher value student math contributions? (Who participates? Does the classroom culture value and encourage most students to speak, only a few, or only the teacher? Who holds authority for knowing mathematics? Do some students hold more status than others?)	
Are students using appropriate mathematic vocabulary?	
What questions do you have?	

* Please note these notes will be collected

Appendix H: Bostic, Matney, and Sondergeld (2017) Mathematical Practice 3 Look-Fors

Construct viable arguments and critique the reasoning of others	
SMP 3 Teacher Indicators	**Teaching Behaviors**
A. Provide and orchestrate opportunities for students to *listen* to the solution strategies of others, *discuss* alternative strategies or solution(s), and defend their ideas	
B. *Ask* higher-order questions which encourage students to defend their ideas, consider student(s) response(s)	
C. *Provide* prompts/tasks that encourage students to think critically about the mathematics they are learning, must be related to argumentation or proving events.	
D. Engage students *in proving events* that encourage students to develop and refine mathematical arguments (including conjectures) or proofs.	

Appendix I: Reflection of Field Observations

Reflection of Field Observation
Date:
1. What did you notice about how the teacher engaged students in Mathematical Practice 3- Construct viable arguments and critique the reasoning of others?
2. Did anything make an imprint or change your thinking about teaching and engaging students in Mathematical Practice #3? Do you have any questions or wonderings about how to engage students in Mathematical Practice #3?
2. In your own words describe what Mathematical Practice #3 would look like in your grade level.
3. Based on your observation notes, how might you plan to engage students in your classroom in Mathematical Practice #3? (This is the first step in your planning portfolio assignment)

Appendix J: Mathematics Learning Autobiography Assignment

When recording your autobiography you may want to consider the following guiding questions (you are not bound by these questions, you may address and describe other experiences that influenced your math interest):

- Was math easy for you?
- Did you always like/not like it?
- What experiences made it easier/harder for you to learn math?
- How did your learning of and/or interest in math vary as you went through school (elementary, middle, high school, college)?
- How was your math learning supported at home?
- In what ways were you alike or different than the other students in your math classes?
- Were most students in the math classes of the same ethnicity, race, gender, linguistic, or socio-economic background as you?
- Was there some person or event that got you interested/turned you off from math?
- How has math affected your career path (including undergraduate major and/or other profession)?

I understand that this could be a very personal topic for many. Please be assured that your video will not be watched or viewed by anyone but me. You may reveal as much or as little as you are comfortable sharing.

Reflection

Reflect upon how your experience learning math might inform your teaching of mathematics to children. You may consider the following guiding questions:

- How do I develop deep understandings of my students' multiple identities so that I can both support and sustain them via mathematics instruction?
- How do I nurture positive and resist negative math identities for students in my mathematics instructional practices, e.g., modeling?
- To what extent do I make mathematics instruction relevant to my students' multiple identities, lived experiences, and cultural practices?
- What are my beliefs as a teacher of Mathematics?

This reflection should be between 300-500 words.

Appendix K: Mathematics Teaching Platform Assignment

This is a statement that captures your beliefs as a mathematics teacher of elementary school students. Your platform should reflect on how and why you plan to teach students mathematics based on what you have learned in Math I and Math II. Look across your weekly reflections where you wrote about connections to your teaching practice. These ideas are a good indicator as to how you will teach math. Use them in your final teaching platform.

Guiding Questions:

- What is mathematics?
- What are math teachers hired to accomplish?
- What personal experiences/stories influence your beliefs?
- How do children learn mathematics and how does this affect your beliefs?
- How would you describe what it means to "do mathematics"? How will you create a classroom environment for doing mathematics?
- What learning theories align with how you plan to teach mathematics? How will you achieve this? (For example, using collaborative learning strategies to align with social constructivist theory or manipulatives and using hands on activities that align with constructivist theories)
- How do you plan to use Mathematics Teaching Practices?
- How will you engage your students in the Standards for Mathematical Practices?
- How will you attend to the rigor of the Math Standards?
- How will you incorporate technology into your lesson?
- How will you teach math equitably to your students?
- What role will problem solving play in your math class?

Your textbook can (and should) be a valuable resource for you particularly chapters 1-7. You can either add to your existing teaching platform or add a tab to your website for a separate Math Teaching Platform(if you do this please have a separate section with clear heading/title). Your Math platform should be posted to your website so that you can submit your website URL for the assignment.

Appendix L: Observation Lesson Reflection Questions

1. Did your lesson go as planned? What changed? Why?
2. Do all students have access to your math lesson (can all students be successful)? Why or why not?
3. How do you recognize that there may be multiple strategies to solve the same problem? Do you value all contributions from students?
4. Describe how your pre-conference supported you in planning your lesson.
5. Describe any additional information that you would have liked to address in your preconference to prepare you to teach this lesson?
6. Describe how your CT supported you in planning your lesson.
7. Use your video to answer the following questions (reference specific times during video):
    a. Describe how you engaged your students in discussion: What kinds of questions did you ask? What higher order thinking questions did you ask? How did you prompt/encourage students to think deeply about the math they were doing? How did you respond to students? Were all students involved in the discussion(s)? Why or why not?
    b. Describe the ways in which you provided opportunity for all students develop collective understanding of the mathematics being taught in your lesson. In what ways did students explain their thinking either to you or with their peers? In what ways did students listen to others explain their thinking? In what ways did students defend their ideas? Were all students involved in these opportunities? Why or why not?
    c. Describe the ways the task(s) supported students in thinking critically about the math they were doing.
8. What have you learned from this observation cycle that you will apply in teaching future math lessons?

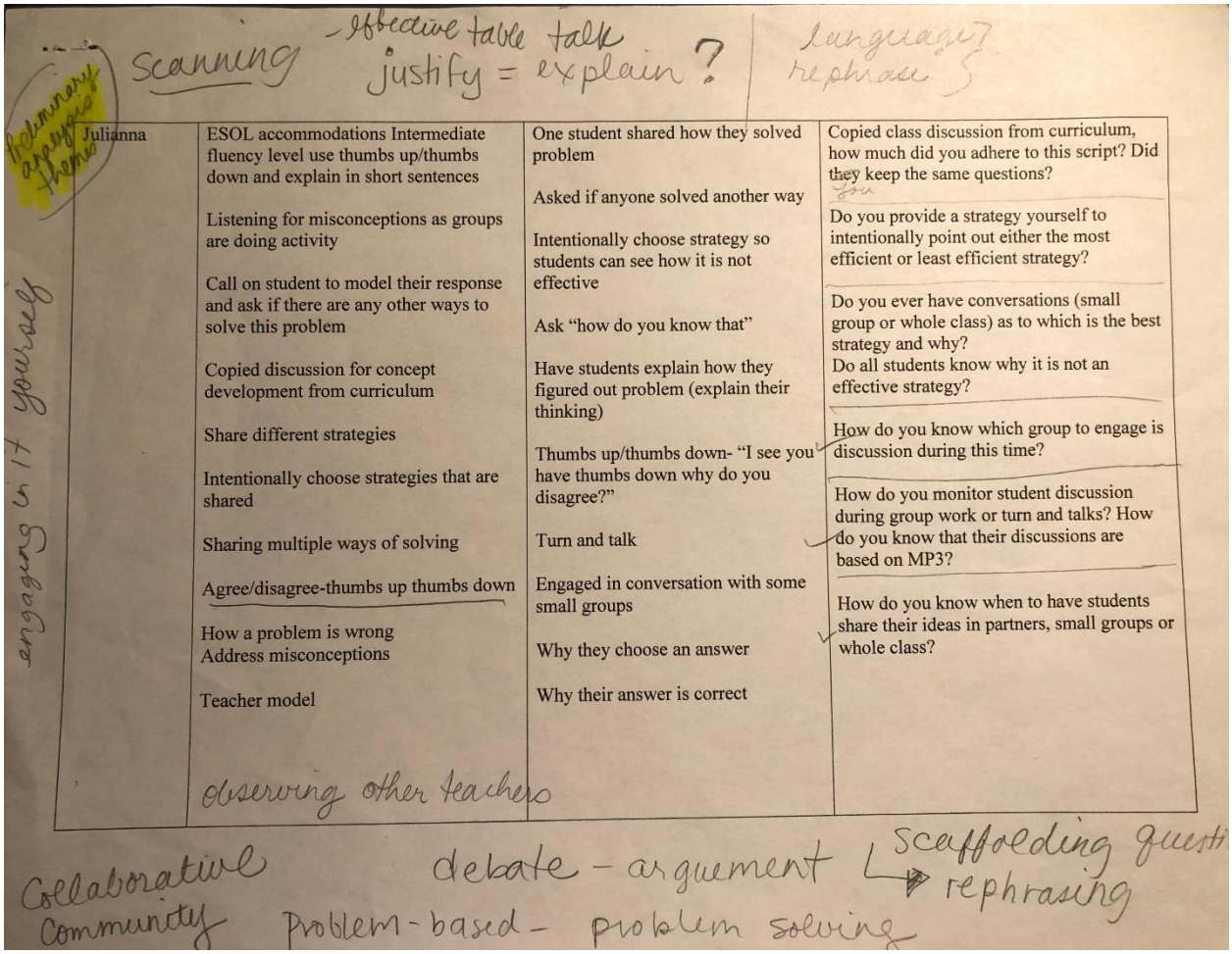
Appendix M: Mathematical Practice 3 Planning Portfolio

Please explain mathematical practice #3 in your own words for your grade level:

Time 1
1. What do you plan to do to facilitate student engagement in mathematical practice #3?
2. What did you actually do when in the classroom?
3. Reflect on your actions. What was the impact of your actions on student engagement in MP#3 and on student learning? What did you learn about your students' mathematical thinking? Did all students have a chance to decide if their answers were right/wrong and explain why? Did all students have a chance to decide of their peers answers were reasonable?

Time 2
1. What do you plan to do to facilitate student engagement in mathematical practice #3?
2. What did you actually do when in the classroom?
3. Reflect on your actions. What was the impact of your actions on student engagement in MP#3 and on student learning? What did you learn about your students' mathematical thinking? Did all students have a chance to decide if their answers were right/wrong and explain why? Did all students have a chance to decide of their peers answers were reasonable?

Appendix N: Informed Interview Questions



Preliminary analysis themes

scanning

- effective table talk

justify = explain?

language? rephrase?

Julianna	ESOL accommodations Intermediate fluency level use thumbs up/thumbs down and explain in short sentences Listening for misconceptions as groups are doing activity Call on student to model their response and ask if there are any other ways to solve this problem Copied discussion for concept development from curriculum Share different strategies Intentionally choose strategies that are shared Sharing multiple ways of solving Agree/disagree-thumbs up thumbs down How a problem is wrong Address misconceptions Teacher model	One student shared how they solved problem Asked if anyone solved another way Intentionally choose strategy so students can see how it is not effective Ask "how do you know that" Have students explain how they figured out problem (explain their thinking) Thumbs up/thumbs down- "I see you have thumbs down why do you disagree?" Turn and talk Engaged in conversation with some small groups Why they choose an answer Why their answer is correct	Copied class discussion from curriculum, how much did you adhere to this script? Did they keep the same questions? Do you provide a strategy yourself to intentionally point out either the most efficient or least efficient strategy? Do you ever have conversations (small group or whole class) as to which is the best strategy and why? Do all students know why it is not an effective strategy? How do you know which group to engage is discussion during this time? How do you monitor student discussion during group work or turn and talks? How do you know that their discussions are based on MP3? How do you know when to have students share their ideas in partners, small groups or whole class?

engaging in it yourself

observing other teachers

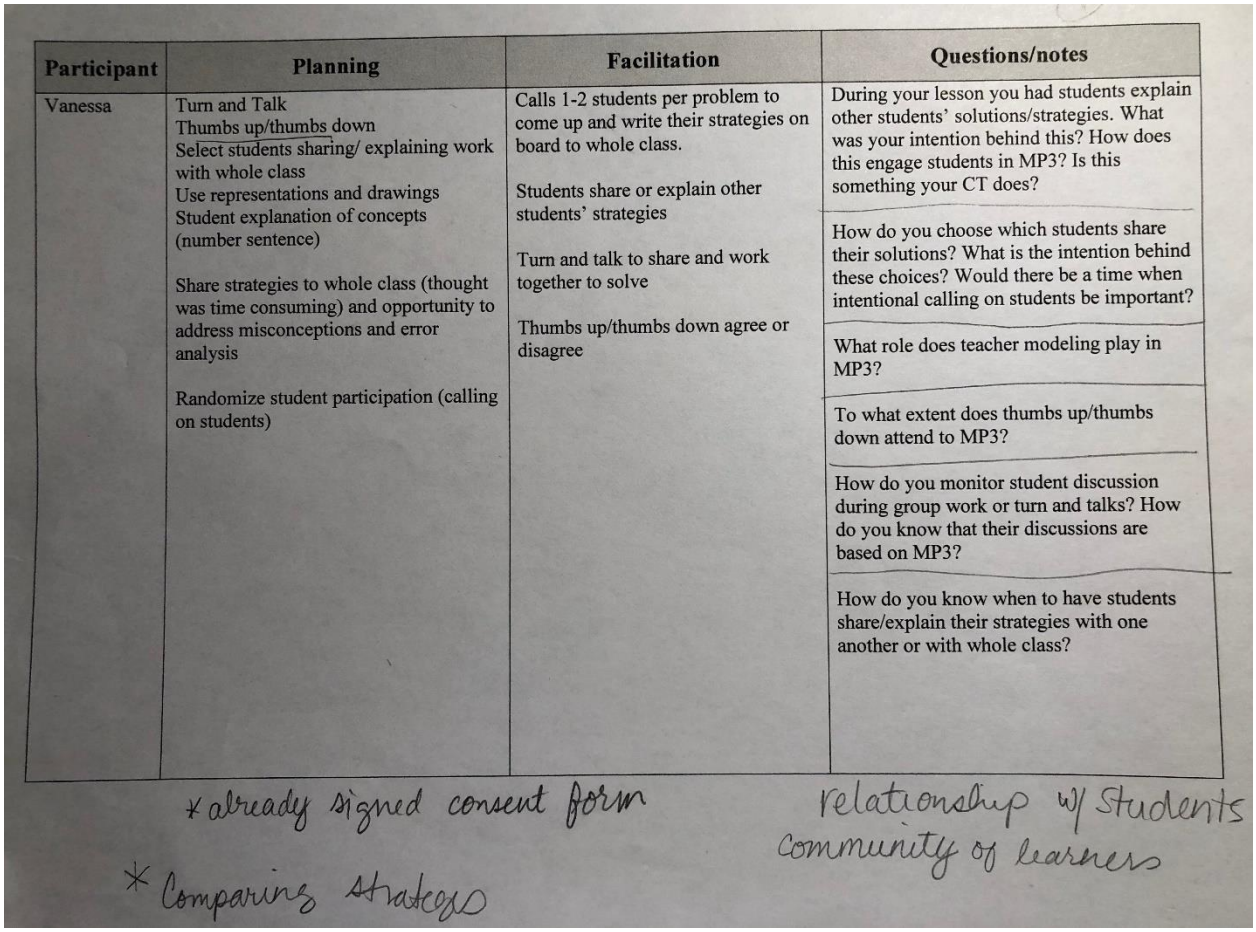
Collaborative Community

debate - argument

Problem-based - problem solving

scaffolding questi

rephrasing



Participant	Planning	Facilitation	Questions/notes
Vanessa	Turn and Talk Thumbs up/thumbs down Select students sharing/ explaining work with whole class Use representations and drawings Student explanation of concepts (number sentence) Share strategies to whole class (thought was time consuming) and opportunity to address misconceptions and error analysis Randomize student participation (calling on students)	Calls 1-2 students per problem to come up and write their strategies on board to whole class. Students share or explain other students' strategies Turn and talk to share and work together to solve Thumbs up/thumbs down agree or disagree	During your lesson you had students explain other students' solutions/strategies. What was your intention behind this? How does this engage students in MP3? Is this something your CT does? How do you choose which students share their solutions? What is the intention behind these choices? Would there be a time when intentional calling on students be important? What role does teacher modeling play in MP3? To what extent does thumbs up/thumbs down attend to MP3? How do you monitor student discussion during group work or turn and talks? How do you know that their discussions are based on MP3? How do you know when to have students share/explain their strategies with one another or with whole class?

* already signed consent form

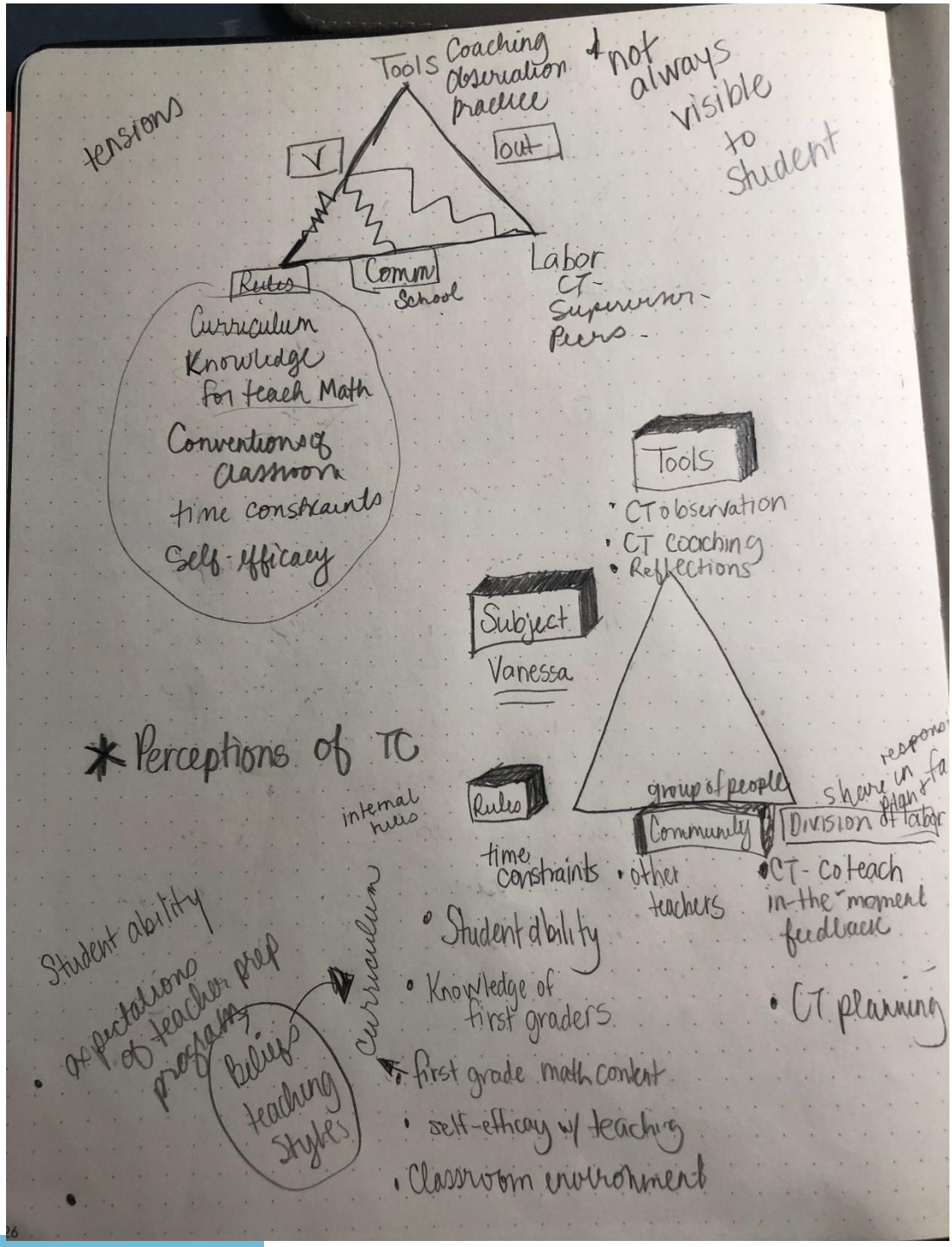
* Comparing strategies

Relationship w/ Students

Community of learners

Kelly	Turn and talk Listening to peers' strategies Displaying student solutions Students explaining how they got their answers (whole class and partners) Asks why after students share answers Opportunity for students to revise answers Allow students to find own mistakes Error analysis- what is wrong about an incorrect problem Student deciding what is right and wrong about their problem or someone else's problem. Teacher intentionally providing these non-examples.	Turn and talk-share how got answer and explain to your partner Called on a few students to share solutions with class. Compare and contrast solutions- does yours look like this? Is yours the same? If its different why is it different?" "Can you share what he said?" "What did you guys do?" "How do we do that?" "Show me" "Why would we start at 8?" "What does this strategy look like?" "How do we say 12 the ten way?" Compare to teachers modeling Thumbs up/thumbs down agree or disagree	During your lesson you had students explain other students' solutions/strategies. What was your intention behind this? How does this engage students in MP3? Is this something your CT does? How do you choose which students share their solutions? What is the intention behind these choices? What role does teacher modeling play in MP3? To what extent does thumbs up/thumbs down attend to MP3? How do you monitor student discussion during group work or turn and talks? How do you know that their discussions are based on MP3? How do you know when to have students share/explain their strategies with one another or with whole class? Process questions (how do we turn into number bond, where do we take 9 from) versus proof questions (how do you know that's right, why did you start at 8, why did you use that strategy, could we have used another way, why or why not) decide

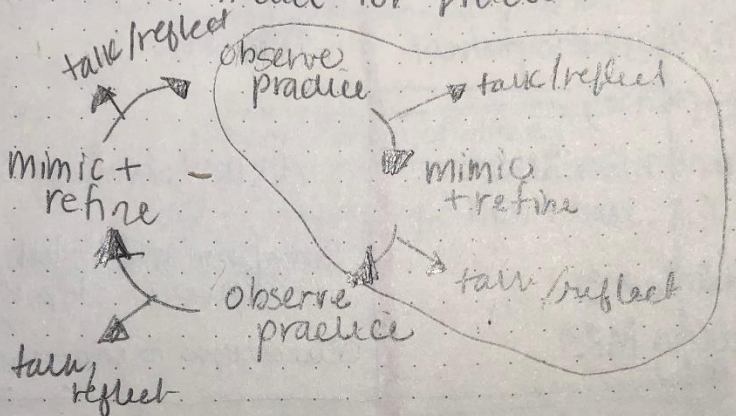
Appendix O: Data Analysis Pictures



Implication – Not something new! We know this!

CTs play a major role for influencing TC practice.

CT model for practice



* Inservice teachers practice can be influenced same way. Providing opportunities for them to see different ways for great teaching

Same process

See someone doing better - try to do what they are doing